1. We introduce the universal quantifier $\forall$ and the existential quantifier $\exists$. A statement of the form $\forall x P(x)$ means that for all $x$ in the universe $P(x)$ is true. A statement of the form $\forall x \in B P(x)$ means that for all $x$ in the set $B$, $P(x)$ is true. A statement of the form $\exists x P(x)$ means that there exists $x$ in the universe $P(x)$ is true. A statement of the form $\exists x \in B P(x)$ means that there exists $x$ in the set $B$ with $P(x)$ true. In mathematics the words ”for some $x$” mean ”there exists $x$”.

2. Quantifier Negation laws:
   $\neg \exists x P(x)$ is equivalent to $\forall x \neg P(x)$.
   $\neg \forall x P(x)$ is equivalent to $\exists x \neg P(x)$.
   $\neg \exists x \in A P(x)$ is equivalent to $\forall x \in A \neg P(x)$.
   $\neg \forall x \in A P(x)$ is equivalent to $\exists x \in \neg P(x)$.

3. The statements $\forall x(x \in A \rightarrow P(x))$ and $\forall x \in A P(x)$ are equivalent. To prove a statement of this form, we begin the proof with ”Suppose $x \in A$. Then we prove $P(x)$.

4. For example, the logical form of the statement $A \subseteq B$ is $\forall x(x \in A \rightarrow x \in B)$ or $\forall x \in A(x \in B)$. To prove that $A \subseteq B$ we begin the proof with ”Suppose $x \in A” and end the proof with ”$x \in B”.

5. When dealing with subsets of the set of real numbers, we sometimes use inequalities to specify the subset. For example, if

$$B = \{x \in \mathbb{R}|x > 0\}$$

then the statement

$$\forall y \in B \exists x \in \mathbb{R}(ax^2 + bx + c = y)$$

can be also denoted by

$$\forall y > 0 \exists x \in \mathbb{R}(ax^2 + bx + c = y).$$
6. Notation. We use the notation $\exists!x$ for the words "there exists a unique $x$.

7. Notation and Axiom. If $A$ is a set, we assume that there exists a set $S$ such that $x \in S$ if and only if $x \subseteq A$. This set is called the power set of $A$ and is denoted by $\mathcal{P}(A)$.

8. Notation and Axiom. Suppose that $S$ is a set, and for each $s \in S$, a set $A_s$ is defined. We assume that there are sets denoted by $\bigcup_{s \in S} A_s$ and $\bigcap_{s \in S} A_s$ such that

$$ (x \in \bigcup_{s \in S} A_s) \leftrightarrow (\exists s \in S(x \in A_s)) $$
and

$$ (x \in \bigcap_{s \in S} A_s) \leftrightarrow (\forall s \in S(x \in A_s)). $$

The set $S$ is called an index set, the family of sets $A_s$ is called an indexed family of sets, the set $\bigcup_{s \in S} A_s$ is called the union of the indexed family of sets, and the set $\bigcap_{s \in S} A_s$ is called the intersection of the indexed family of sets.

9. Notation and Axiom. Suppose that $\mathcal{F}$ is a set, and each element of $\mathcal{F}$ is a set. We assume that there are sets denoted by $\bigcup \mathcal{F}$ and $\bigcap \mathcal{F}$ such that

$$ (x \in \bigcup \mathcal{F}) \leftrightarrow (\exists A \in \mathcal{F}(x \in A)) $$
and

$$ (x \in \bigcap \mathcal{F}) \leftrightarrow (\forall A \in \mathcal{F}(x \in A)). $$

The set $\mathcal{F}$ is called a family of sets, the set $\bigcup \mathcal{F}$ is called the union of the family of sets, and the set $\bigcap \mathcal{F}$ is called the intersection of the family of sets.