1. In these notes we suppose that $R$ is a relation on a set $A$. Also, recall that $xRy$ means $(x, y) \in R$. We summarize some basic definitions.

2. $R$ is reflexive iff $\forall x \in A(xRx)$. 

3. $R$ is symmetric iff $\forall x \in A, \forall y \in A, (xRy \rightarrow yRx)$. 

4. $R$ is transitive iff $\forall x \in A \forall y \in A, \forall z \in A, ((xRy \land yRz) \rightarrow xRz)$. 

5. $R$ is antisymmetric iff $\forall x \in A \forall y \in A, ((xRy \land yRx) \rightarrow x = y)$. 

6. $R$ is a partial order iff $R$ is reflexive, transitive, and antisymmetric. 

7. $R$ is a total order iff $R$ is a partial order and also the following holds: 
   \[ \forall x \in A \forall y \in A (xRy \lor yRx). \]

8. Suppose that $R$ is a partial order on $A$. Suppose that $B \subseteq A$ and $b \in B$. We say that $b$ is a smallest (or $R$-smallest) element of $B$ iff $\forall x \in B(bRx)$. 

9. Suppose that $R$ is a partial order on $A$. Suppose that $B \subseteq A$ and $b \in B$. We say that $b$ is a minimal (or $R$-minimal) element of $B$ iff 
   \[ \neg \exists x \in B (xRb \land x \neq b). \]

10. Suppose that $R$ is a partial order on $A$. Suppose that $B \subseteq A$ and $a \in A$. We say that $a$ is a lower bound for $B$ iff 
    \[ \forall x \in B(aRx). \]

11. Suppose that $R$ is a partial order on $A$. Suppose that $B \subseteq A$ and $a \in A$. We say that $a$ is an upper bound for $B$ iff 
    \[ \forall x \in B(xRa). \]

12. Suppose that $R$ is a partial order on $A$, and $B \subseteq A$. Let $L$ be the set of all lower bounds for $B$. If $L$ has a largest element, then this largest element is called the greatest lower bound of $B$. 

13. Suppose that $R$ is a partial order on $A$, and $B \subseteq A$. Let $U$ be the set of all upper bounds for $B$. If $U$ has a smallest element, then this smallest element is called the least upper bound of $B$.

14. Suppose that $A$ is a set and $\mathcal{F} \subseteq \mathcal{P}(A)$. We say that $\mathcal{F}$ is pairwise disjoint iff every pair of distinct elements of $\mathcal{F}$ are disjoint. We say that $\mathcal{F}$ is a partition of $A$ iff $\mathcal{F}$ is pairwise disjoint, $\bigcup \mathcal{F} = A$, and $\emptyset \notin \mathcal{F}$.

15. Suppose that $R$ is a relation on a set $A$. We say that $R$ is an equivalence relation iff $R$ is reflexive, transitive, and symmetric.

16. Suppose that $R$ is an equivalence relation on $A$. Suppose that $x \in A$. The equivalence class of $x$ denoted $[x]$ is given by

$$[x] = \{y \in A | xRy\}.$$  

We let $A/R$ (in words, $A$ modulo $R$) denote the set of equivalence classes.

17. We have the following theorem: Suppose that $R$ is an equivalence relation on $A$. Then $A/R$ is a partition of $A$.

18. We also have the following theorem: Suppose that $A$ is a set, and $\mathcal{F}$ is a partition of $A$. Then there is an equivalence relation $R$ on $A$ such that $A/R = \mathcal{F}$. 