1. Definition: Suppose that $F$ is a relation from $A$ to $B$. We say that $F$ is a function from $A$ to $B$ iff for every $a \in A$ there is a unique $b \in B$ such that $(a, b) \in F$. We use the notation $F : A \rightarrow B$ to indicate that $F$ is a function from $A$ to $B$. Also, if $a \in A$, we let $F(a)$ denote the unique $b \in B$ such that $(a, b) \in F$.

2. Theorem: Suppose that $f$ and $g$ are functions from $A$ to $B$. Then $f = g$ if and only if $\forall a \in A (f(a) = g(a))$.

3. Theorem: Suppose that $f : A \rightarrow B$ and $g : B \rightarrow C$. Then $g \circ f : A \rightarrow C$ and for every $a \in A$ we have $(g \circ f)(a) = g(f(a))$.

4. Definition and Remark. Suppose that $f : A \rightarrow B$. We say that $f$ is one-to-one iff for all $a_1 \in A$ and $a_2 \in A$ if $f(a_1) = f(a_2)$ then $a_1 = a_2$. We say that $f$ is onto iff for every $b \in B$ there exists $a \in A$ with $f(a) = b$. Note that $f$ is onto if and only if $B$ is the range of $f$.

5. Theorem: Suppose that $f : A \rightarrow B$ and $g : B \rightarrow C$. If $f$ and $g$ are one-to-one, then $g \circ f$ is one-to-one. If $f$ and $g$ are onto, then $g \circ f$ is onto.

6. Remark and Theorem. Suppose that $f : A \rightarrow B$. Then $f$ is also a relation from $A$ to $B$. So the inverse relation $f^{-1}$ is defined and is a relation from $B$ to $A$. We have the following theorem: $f^{-1}$ is a function from $B$ to $A$ if and only if $f$ is one-to-one and onto.

7. Theorem: Suppose that $f : A \rightarrow B$ and $g : B \rightarrow A$. Suppose also that $g \circ f = i_A$ and $f \circ g = i_B$. Then $g = f^{-1}$. 