1. Suppose that \( a, b, c, d \) are real numbers. Prove that if \( b \neq 0 \) and \( d \neq 0 \), then

\[
\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}.
\]

Note: This is Theorem 1.7 part (h) in the notes on real numbers.

Answer:

\[
\frac{a}{b} + \frac{c}{d} = ab^{-1} + cd^{-1} \quad \text{by Theorem 1.7 part (a)}
\]
\[
= ab^{-1}dd^{-1} + cd^{-1}bb^{-1} \quad \text{(using MIV)}
\]
\[
= b^{-1}d^{-1}(ad + bc) \quad \text{(using D)}
\]
\[
= (bd)^{-1}(ad + bc) \quad \text{by Theorem 1.4 part (l)}
\]
\[
= \frac{ad + bc}{bd} \quad \text{by Theorem 1.7 part (a)}
\]

2. Suppose that \( a, b \) are real numbers. Prove that

(i) \( a > 0, b > 0 \) imply \( ab > 0 \).

(ii) \( a > 0, b < 0 \) imply \( ab < 0 \).

(iii) \( a < 0, b < 0 \) imply \( ab > 0 \).

Note: This is Theorem 1.15 part (d) in the notes on real numbers.

Answer:

(i) Suppose that \( a > 0 \) and \( b > 0 \). Starting with the inequality \( a > 0 \), and multiplying by \( b \) we obtain \( ab > 0b \) (by OM). Since \( 0b = 0 \), we have \( ab > 0 \).

(ii) Suppose that \( a > 0 \) and \( b < 0 \). Starting with the inequality \( b < 0 \), and multiplying by \( a \) we obtain \( ab < 0a \) (by OM). Since \( 0a = 0 \), we have \( ab < 0 \).

(iii) Suppose that \( a < 0 \) and \( b < 0 \). Then \( -a > 0 \) and \( -b > 0 \) by 1.15 part b. By part (i) above we have \((-a)(-b) > 0 \). Since \( ab = (-a)(-b) \) we have \( ab > 0 \).