MHF 3202, Dr. Block, Sample Final Exam, Fall 2019

There are 11 problems worth a total of 50 points.

1. (5 points) Construct a truth table for the formula \((P \land \sim Q) \Rightarrow R\).

2. (3 points) Negate the following statement.

   For every integer \(x\), if 9 divides \(x^2 - 7x\), then 3 divides \(x\).

3. (3 points) Determine if the statement is true or false.

   \[ \exists x \in \mathbb{R}, \forall y \in \mathbb{R}, x + 2y = 5 \]

4. (3 points) Determine if the statement is true or false. The logical formulas \(P \lor Q\) and \(\sim P \Rightarrow Q\) are equivalent.

5. (3 points) Determine if the statement is true or false. For any set \(A\), we have \(A \subseteq \mathcal{P}(A)\).

6. (3 points) Determine if the statement is true or false. Suppose that \(R_1\) and \(R_2\) are relations on \(A\). If \(R_1\) and \(R_2\) are transitive, then \(R_1 \cap R_2\) is transitive.

7. (3 points) Determine if the statement is true or false. Let \(A = \mathcal{P}(\mathbb{R})\), and define \(f : \mathbb{R} \to A\) by the formula \(f(x) = \{y \in \mathbb{R} : y^2 < x\}\). Then \(f\) is injective.

8. (6 points) Prove that if \(A\), \(B\), and \(C\) are sets, then

   \[ A - (B \cup C) = (A - B) \cap (A - C). \]

9. (6 points) Prove that for any sets \(A\) and \(B\), if \(\mathcal{P}(A) \cup \mathcal{P}(B) = \mathcal{P}(A \cup B)\) then either \(A \subseteq B\) or \(B \subseteq A\).

10. (6 points)

    Suppose that \(X\) is a nonempty set. Let \(A\) denote the set of functions from \(X\) to \(X\). Let \(B\) denote the set of \(h \in A\) such that \(h\) is bijective. Define a relation \(R\) on \(A\) by

    \[ R = \{(f, g) \in A \times A : \exists h \in B \text{ such that } h \circ f = g \circ h\}. \]

    Prove that \(R\) is an equivalence relation on \(A\).
11. (6 points) Let $A = \mathbb{R} - \{2\}$, and let $B = \mathbb{R} - \{3\}$. Let $f : A \to B$ be defined by the formula $f(x) = \frac{3x}{x-2}$. Prove that $f$ is bijective. Also, find a formula for $f^{-1}$.

12. (6 points) Prove the following by mathematical induction: For all $n \in \mathbb{N}$,

$$9 \mid (4^n + 6n - 1).$$

Recall that this notation means that 9 divides $(4^n + 6n - 1)$.