MHF 3202, Dr. Block, Quiz 3, Fall 2019

1. (3 points) Write the following as an English sentence. Say whether it is true or false.

\[ \forall X \subseteq \mathbb{N}, \exists n \in \mathbb{Z}, \vert X \vert = n \]

Answer: For every subset \( X \) of \( \mathbb{N} \) there exists an integer \( n \) such that the cardinality of \( X \) is \( n \).

The statement is false.

2. (3 points) Translate the following sentence into symbolic logic.
   If \( x \) is a rational number and \( x \neq 0 \), then \( \tan(x) \) is not a rational number.

Answer: \(( (x \in \mathbb{Q}) \land (x \neq 0)) \Rightarrow (\tan(x) \notin \mathbb{Q})\)

3. (4 points) Negate the following sentence.
   For every positive number \( \epsilon \), there is a positive number \( M \) for which \( \vert f(x) - b \vert < \epsilon \) whenever \( x > M \).

Answer: There exists a positive number \( \epsilon \) such that for every positive number \( M \) there is some \( x > M \) with \( \vert f(x) - b \vert \geq \epsilon \).