MHF 3202, Dr. Block, Sample Exam 1 with answers
  There are 8 problems worth a total of 40 points.

1. (4 points) Let R stand for the statement "it is raining," S stand for the statement "it is snowing," and C stand for the statement "the game has been canceled." Express the following sentence as a logical formula.

   If the game has been canceled, then either it is raining or it is snowing.

   Answer:
$$C \rightarrow (R \vee S)$$

2. (6 points) Construct a truth table for the formula $(P \wedge \neg Q) \rightarrow R$.

   Answer:

P	Q	R	$\neg Q$	$P \wedge \neg Q$	$(P \wedge \neg Q) \rightarrow R$
T	T	T	F	F	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	T	F	F	F	T
F	F	T	T	F	T
F	F	F	T	F	T

3. (6 points) Use the laws of logic stated in the text to prove that the formulas $(P \vee \neg R) \wedge (Q \vee \neg R)$ and $\neg((\neg(P \wedge Q)) \wedge R)$ are equivalent.

   Use one law at a time and state the name of the law you are using in each step.

   Answer:

$$
\begin{aligned}
\neg((\neg(P \wedge Q)) \wedge R) &\equiv \neg\neg(P \wedge Q) \vee \neg R && \text{DeMorgan} \\
&\equiv (P \wedge Q) \vee \neg R && \text{Double Negation} \\
&\equiv \neg R \vee (P \wedge Q) && \text{Commutative} \\
&\equiv (\neg R \vee P) \wedge (\neg R \vee Q) && \text{Distributive} \\
&\equiv (P \vee \neg R) \wedge (Q \vee \neg R) && \text{Commutative}
\end{aligned}
$$

4. (5 points) Find a formula which uses only the connectives $\wedge$, $\vee$ and $\neg$ which is equivalent to $P \to (Q \to R)$.

Answer:
$$\neg P \vee (\neg Q \vee R)$$

5. (4 points) Draw a Venn diagram for the sets A, B, and C, and shade in the region on diagram which corresponds to $(A \cup B) \smallsetminus (C \smallsetminus B)$.

Answer: I will post a Venn diagram on a separate link.

6. (5 points) Let P, Q, R represent the statements $x \in A$, $x \in B$, $x \in C$, respectively. Express the statement $x \in A \cap (B \smallsetminus C)$ as a logical formula in terms of P, Q, R.

Answer:
$$P \wedge (Q \wedge \neg R)$$

7. (5 points) Negate the following statement and then reexpress the result as a positive statement.
$$\exists x \forall y [y > x \to \exists z (z^2 + 5z = y)]$$

Answer:
$$\forall x \exists y [(y > x) \wedge \forall z (z^2 + 5z \neq y)]$$

8. (5 points) Let I be the index set $\{1, 2, 3\}$. For each $i \in I$, let
$$A_i = \{i, i + 1, i + 2, i + 5\}.$$

List the elements of the following sets:

$\bigcup_{i \in I} A_i$

$\bigcap_{i \in I} A_i$

Answer:

$\bigcup_{i \in I} A_i = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$\bigcap_{i \in I} A_i = \{3\}$