

**MHF 3202, Dr. Block, Sample Exam 1, Spring 2020**

There are 7 problems worth a total of 40 points.

1. (5 points) Write the following set by listing the elements of the set between braces.

$$\mathcal{P}(\{1, 2\}) \times \mathcal{P}(\{3\})$$

2. (6 points) Construct a truth table for the formula  $(P \wedge \sim Q) \Rightarrow R$ .

3. (6 points) Negate the following sentence.

For every  $\epsilon > 0$  there exists  $\delta > 0$  such that if  $|x - 5| < \delta$ , then  $|x^2 - 25| < \epsilon$ .

4. (2 points) Determine whether the following statement is true or false.

$$\exists a \in \mathbb{Z}, \forall b \in \mathbb{Z}, a + b = 0$$

5. (7 points) Prove the following using a direct proof.

If  $x \in \mathbb{R}$  and  $0 < x < 4$ , then  $\frac{4}{x(4-x)} \geq 1$ .

6. (7 points) Prove the following with contrapositive proof.

Suppose  $a \in \mathbb{Z}$ . If  $a^2$  is not divisible by 4, then  $a$  is odd.

7. (7 points) Prove the following using either direct proof or contrapositive proof.

If  $a \in \mathbb{Z}$  and  $a \equiv 1 \pmod{5}$ , then  $a^2 \equiv 1 \pmod{5}$ .