MHF 3202, Dr. Block, Sample Exam #2

There are six problems worth seven points each.

1. Let $a, b, c, d$ be real numbers such that $0 < a < b$ and $d > 0$. Prove that if $ac \geq bd$ then $c > d$.

2. Prove that for every real number $x$, if $x \neq 0$, then there is a unique real number $y$ such that for every real number $z$ we have $zy = z/x$.

3. Prove that for every real number $x$ such that $x > 2$ there is a real number $y$ such that $y + \frac{1}{y} = x$.

4. Suppose that $\mathcal{F}$ and $\mathcal{G}$ are nonempty families of sets such that every element of $\mathcal{F}$ is a subset of every element of $\mathcal{G}$. Prove that $\bigcup \mathcal{F} \subseteq \bigcap \mathcal{G}$.

5. Prove that $\forall x \in \mathbb{R}((\exists y \in \mathbb{R} (x + y = xy)) \iff x \neq 1)$.

6. Let $A, B, C$ be sets such that $A \cap C \subseteq B \cap C$ and $A \cup C \subseteq B \cup C$. Prove that $A \subseteq B$. 