1. Use the method of proof by contradiction to prove the following.
   There exist no integers \(a\) and \(b\) for which \(18a + 6b = 1\).

2. Suppose that \(A, B,\) and \(C\) are sets. Prove that if
   \[ A - C \subseteq B - C. \]
   then
   \[ (A \cup C) \subseteq (B \cup C). \]

3. Prove that \(\{12a + 4b : a, b \in \mathbb{Z}\} = \{4c : c \in \mathbb{Z}\}\).

4. Suppose that \(a, b, p \in \mathbb{Z}\) and \(p\) is prime. Prove that if \(p | ab\), then \(p | a\) or \(p | b\).

5. Prove the following statement by mathematical induction.
   For every integer \(n \geq 0\),
   \[ 9 | (4^{3n} + 8). \]