MHF 3202, Dr. Block, Sample Final Exam

There are 11 problems worth a total of 50 points.

1. (5 points) Construct a truth table for the formula $(P \land \neg Q) \rightarrow R$.

2. (3 points) Determine if the statement is true or false.
   \[
   \forall x \in \mathbb{N} \exists y \in \mathbb{N} (x = 2y)
   \]

3. (3 points) Determine if the statement is true or false. If $\mathcal{F}$ is a family of sets and $A \in \mathcal{F}$, then $A \subseteq \cup \mathcal{F}$.

4. (3 points) Determine if the statement is true or false. For any set $A$, we have $A \subseteq \mathcal{P}(A)$.

5. (3 points) Determine if the statement is true or false. Suppose that $R_1$ and $R_2$ are relations on $A$. If $R_1$ and $R_2$ are transitive, then $R_1 \cap R_2$ is transitive.

6. (3 points) Determine if the statement is true or false. Let $A = \mathcal{P}(\mathbb{R})$, and define $f : \mathbb{R} \rightarrow A$ by the formula $f(x) = \{y \in \mathbb{R} | y^2 < x\}$. Then $f$ is one-to-one.

7. (6 points) Prove that $\forall x \in \mathbb{R}[(\exists y \in \mathbb{R} (x + y = xy)) \leftrightarrow x \neq 1]$.

8. (6 points) Prove that for any sets $A$ and $B$, if $\mathcal{P}(A) \cup \mathcal{P}(B) = \mathcal{P}(A \cup B)$ then either $A \subseteq B$ or $B \subseteq A$.

9. (6 points) Suppose $R$ is a relation from $A$ to $B$ and $S$ and $T$ are relations from $B$ to $C$. Prove that
   \[
   (S \circ R) \setminus (T \circ R) \subseteq (S \setminus T) \circ R.
   \]

10. (6 points) Let $f : \mathbb{R} \setminus \{2\} \rightarrow \mathbb{R} \setminus \{3\}$ be defined by the formula $f(x) = \frac{3x}{x-2}$. Prove that $f$ is one-to one and onto.

11. (6 points) Prove the following by mathematical induction: For all $n \in \mathbb{N}$,
   \[
   64 | (9^n - 8n - 1).
   \]
   Recall that this notation means that $(9^n - 8n - 1)$ is an integer multiple of 64.