Solutions for the Practice 2nd Half-final

1. \( x = (7/4)e^t + (7/4)e^{-t} - (3/2)\cos t \) and \( y = (7/4)e^t - (7/4)e^{-t} - (1/2)\cos t \).

2. We get that \( a_0 \) and \( a_1 \) are free and \( a_2 = a_1 / 2 \), and

\[
a_{n+2} = \frac{a_{n-1}}{(n+2)(n+1)} + \frac{a_{n+1}}{n+2}.
\]

3. 

\[
y(t) = e^{-2t} \sin t + u(t - \pi) \sin(t - \pi)e^{-2(t-\pi)}
\]

\[
= e^{-2t} \sin t - e^{-2(t-\pi)}u(t - \pi) \sin t.
\]

4. (a) True, since \( \lim_{\infty} a_n / a_{n+1} = \lim_{\infty} n + 1 = \infty \).

(b) No, 0 is an ordinary point. Use the L’Hospital rule.

(c) No, that is only true in regular singular points.