Review for Exam #2

1. Exam will be posted as pdf on the LADS HW page around 2:00pm on Wednesday, March 25
2. You should do the exam in ~~your~~ length to be decided then scan and send it to me via email.
3. I must receive it by 5:00pm on Thursday March 26.
4. The exam is open book, notes, review materials
5. You can use a calculator or computer program but you must show all intermediate steps in your "by hand" calculations, so electronic assist might slow you down
6. There will be two types of problem
   (a) "Show that problems" from the list below, so prepare them and have your notes ready
   (b) "Hand calculations" of the type given below, so you should be able to do these quickly, so practise.
7. I will provide soln to the review "hand calculations" but not the "show that" ones.

Several of these problems will appear verbatim on the exam

(0) All of the "show that" problems from HW 5, 6, 7, 8.

(1) State the Spectral Theorem

(2) Deive the normal equations for the least squares solution to $A\vec{x}=\vec{b}$ where A is $m\times n$, $m\geq n$ and $\operatorname{rank}(A)=n$

(3) ~~[illegible]~~ A is $m\times n$, $m\geq n$, $\operatorname{rank}(A)=n$, show using the SVD of A, that A^TA is invertible by giving a formula for its inverse

(4) If $\operatorname{rank}(A)=r$, how many nonzero singular values does A have?

(5) If $A=U\Lambda V^T$, the thin SVD, give a formula for the matrix B which minimizes $\|B-A\|$ over all rank one matrices B.

(6) If $A=QBQ^T$ with Q orthogonal, show that A and B have the same singular values

(7) If Q is $m\times n$, $m\geq n$ and has orthonormal columns, show that $Q^TQ=I$. Prove or disprove that $QQ^T=I$.

(8) If the eigenvalues of A^TA are $\lambda_1,\dots,\lambda_n$ the singular values of A are ?

There will be hand calculation problems of this sort as well as like any of the hand calculation problems from HW 5, 6, 7, 8

(1) Let $Q = \begin{bmatrix} 1/2 & 1/2 \\ -1/2 & 1/2 \\ 1/2 & 1/2 \\ -1/2 & 1/2 \end{bmatrix}$ Find a matrix P so that $P\vec{b}$ is the orthogonal projection of $\vec{b}$ onto $\mathrm{col}(Q)$.

(2) You are given that the matrix A has SVD

$$A = \begin{bmatrix} 1/\sqrt{3} & -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{3} & 0 & 0 \\ 1/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 2\sqrt{6} & 0 & 0 \\ 0 & \sqrt{6} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{3} & 1/\sqrt{2} \\ 0 & 1/\sqrt{3} & 0 \\ 1/\sqrt{2} & 1/\sqrt{3} & 1/\sqrt{2} \end{bmatrix}.$$

Find the best rank 1 approximation to A in the 2-norm and the best rank 2 approximation. Your answer must be a fully computed matrix, ~~with~~

(3) Let $A = \begin{bmatrix} 1 & -1 \\ 1 & -1 \\ 1 & 5 \\ 1 & 5 \end{bmatrix}$. Find the thin QR decomposition for A and use it to find the least squares solution to $A\vec{x}=\vec{b}$ with $\vec{b} = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}$ by computing the pseudo inverse A^+

(4) Compute by hand the SVD of

$$A = \begin{bmatrix} 3 & 4 \\ 0 & 5 \end{bmatrix}$$

What is the 2-norm condition number of A

~~What is the Frobenius norm condition number of A.~~