L1

1

BASIC DATA OBJECTS

Vectors - one-dimensional array

$$\vec{V} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} \in \mathbb{R}^n$$

$\in$ "in"

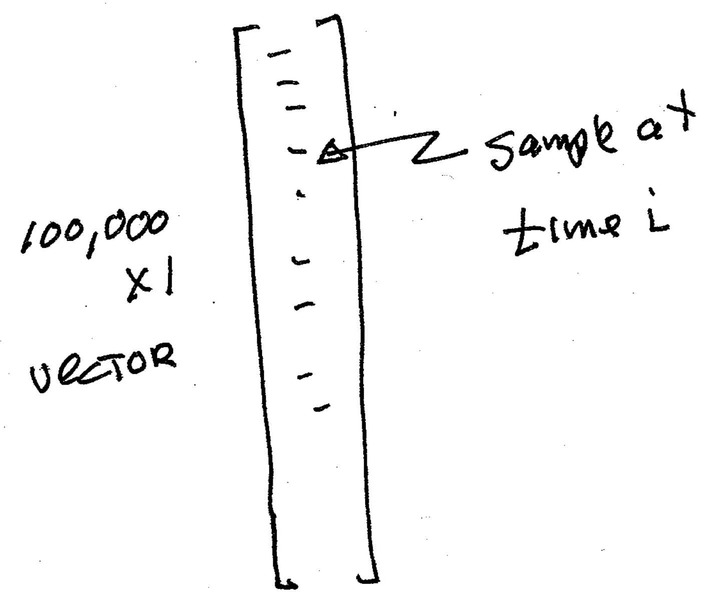
$\mathbb{R}^n$ all n dimensional real vectors

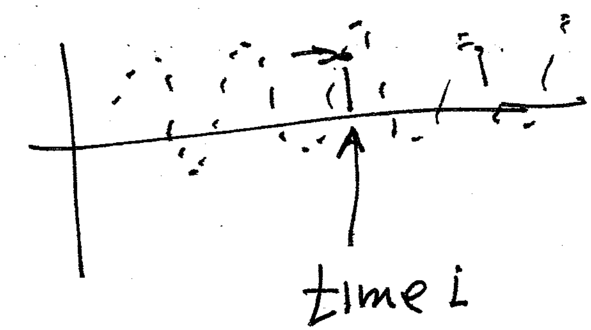
- All vectors are by default <u>column</u> vectors in this course (maybe not in Matlab, etc)
- a row vector is $\vec{V}^T = [v_1 \cdots v_n]$

T Transpose

Examples

(1) A PCM (Pulse Code Modulation) file is a raw digital audio file and each element of the vector is the amplitude at a sample time





(2) The price of all listed stocks on the DJ at a given time

$$\begin{bmatrix} 2.73 \\ 192 \\ \vdots \\ \\ \end{bmatrix}$$

(3) position and velocities of all nine planets write now $9 \times 3 \times 3 = 54 \times 1$ vector

MATRICES OR ARRAYS

$$A = m\left\{\overbrace{\begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & & & \\ \vdots & & & \\ A_{m1} & & & A_{mn} \end{bmatrix}}^{n}\right.$$

$m \times n$ matrice

$$\begin{array}{cc} & j \\ & \downarrow \\ i \rightarrow & \begin{bmatrix} A_{ij} \end{bmatrix} \end{array}$$

Row i column j element is denoted A_{ij}

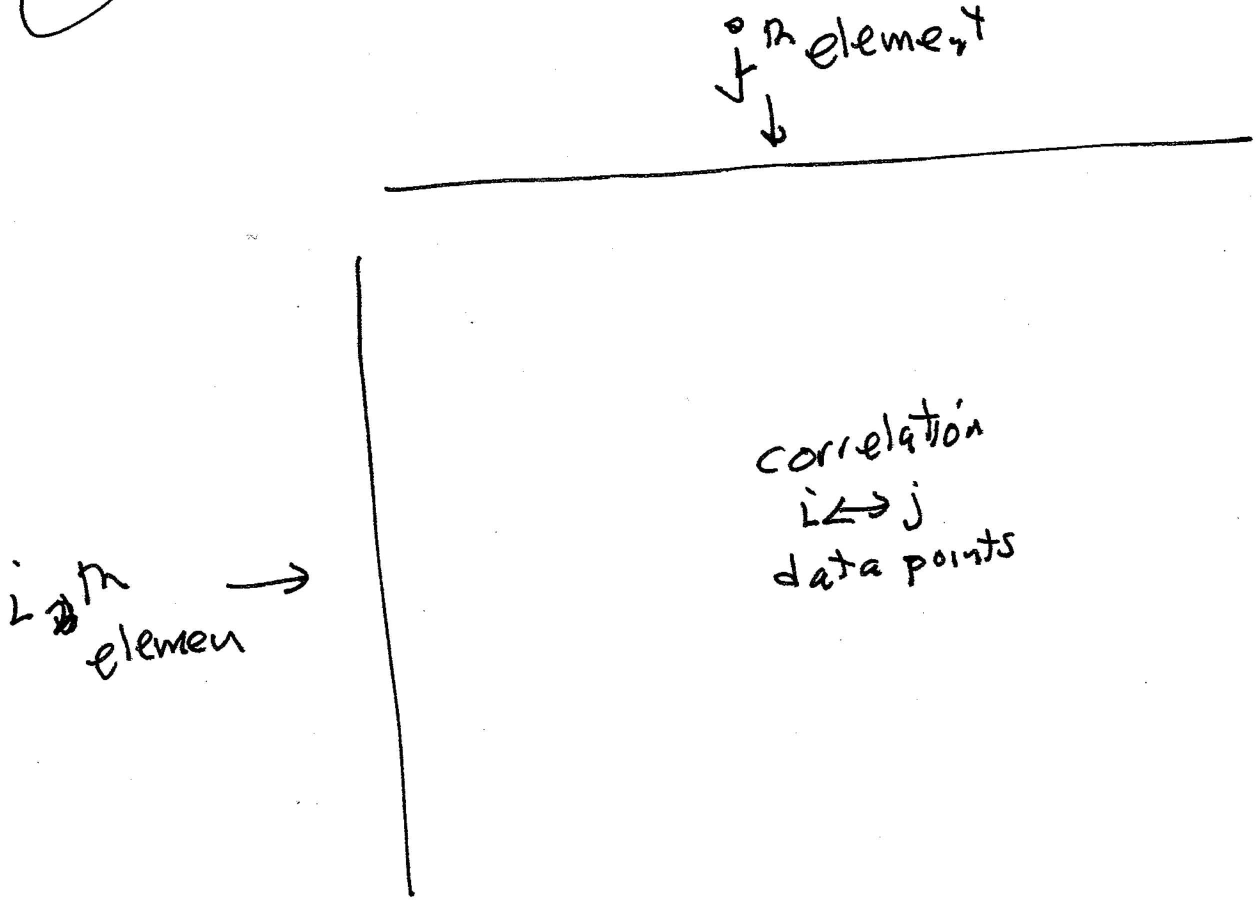
Examples

(1) grey scale image 640 x 1024 is a 640 x 1024 with integer entries $0 \le A_{ij} \le 255$ with element A_{ij} telling you how dark the pixel at position ij is

(2) Contain a list of data vectors, say column is the price of all listed stocks at sample time j

$$\begin{bmatrix} \vec{v}_1 & \vec{v}_2 & \cdots & \vec{v}_j & \cdots & \vec{v}_n \end{bmatrix} \qquad v_j = \begin{bmatrix} - \\ - \\ - \\ - \\ - \\ - \\ - \\ - \\ - \\ - \end{bmatrix}$$

(3) Correlations between two data vectors



(4) Transformations (Linear) from one data vector to another

$$L(\vec{v}) = A\vec{v}$$

$$\vec{v} \xrightarrow{L} A\vec{v}$$

<u>DEF</u>: If L transforms one vector spce into another it is called <u>Linear</u> if

(1) $L(\vec{v}+\vec{w}) = L(\vec{v}) + L(\vec{w})$ [eg filter]

(2) $L(\alpha\vec{v}) = \alpha L(\vec{v})$

<u>Theorem</u>: For any Linear transformation there is a matrix with $L(\vec{v}) = A\vec{v}$.

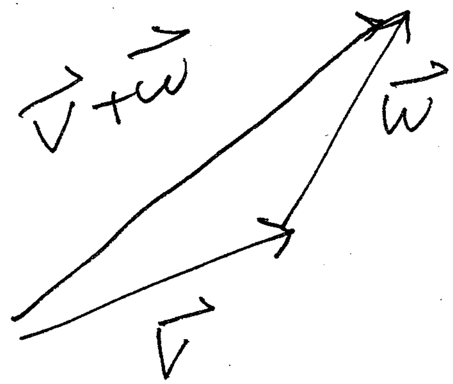
Review

$$\underline{v} = \vec{v} = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix} = \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix}$$

$\boxed{\text{Vectors are column vectors}}$

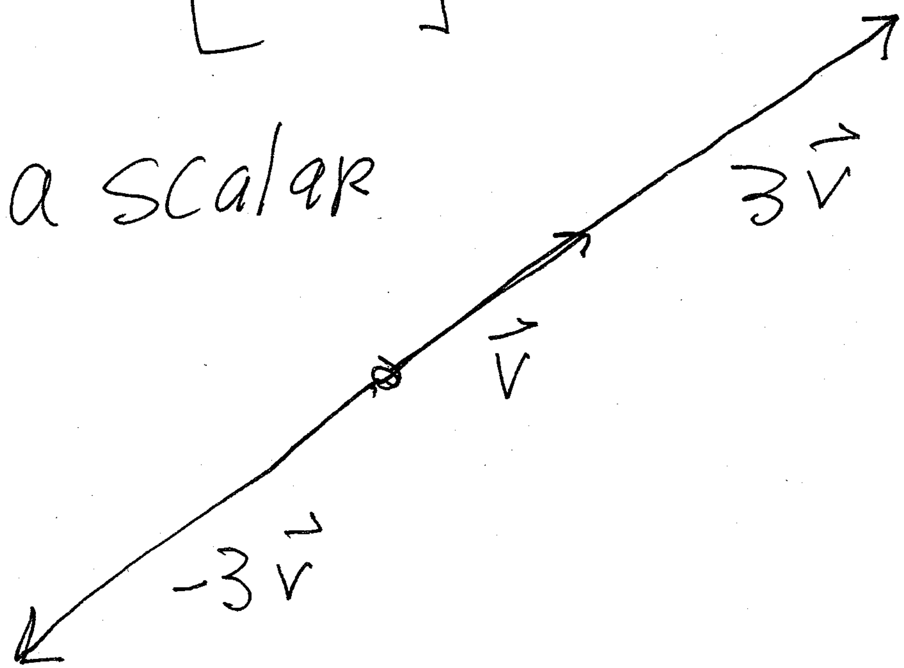
ie $n \times 1$ array.

$$\vec{v} + \vec{w} = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix} + \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix} = \begin{bmatrix} v_1 + w_1 \\ \vdots \\ v_n + w_n \end{bmatrix}$$



<u>Rescaling</u> $\alpha\vec{v} = \begin{bmatrix} \alpha\vec{v}_1 \\ \vdots \\ \alpha\vec{v}_n \end{bmatrix} \quad \alpha \in \mathbb{R}$

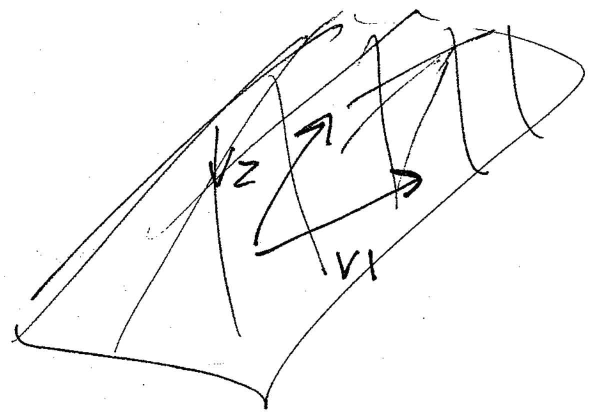
α is called a scalar



$\boxed{\text{Linear combination}}$

$$\alpha_1 \vec{v}_1 + \dots + \alpha_n \vec{v}_n = \sum_{i=1}^{n} \alpha_i \vec{v}_i$$

$\text{Span}(\vec{v}_1, \dots, \vec{v}_n)$ = all linear combinations of the vectors



$$\text{Span}\left(\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}-2\\1\\4\end{bmatrix}\right)$$

$= \text{plane in } \mathbb{R}^3$

$\boxed{\text{Inner product}}$

$$\vec{u} \cdot \vec{v} = u_1 v_1 + \dots + u_n v_n$$

$$= \sum_{i=1}^{n} u_i v_i$$

Also written $\langle \vec{u}, \vec{v} \rangle$

As vectors in multiplication

$$\vec{u}^T \vec{v} = \begin{bmatrix} u_1 & u_2 & \dots & u_n \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$$

$\mathcal{B} = \{\vec{v}_1, \ldots, \vec{v}_n\}$ in $\mathbb{R}^n$ is a basis if any vector $\vec{w} \in \mathbb{R}^n$ can be written in a <u>unique</u> way as

$$\vec{w} = \alpha_1 \vec{v}_1 + \cdots + \alpha_n \vec{v}_n$$

The α_i are the coordinates of $\vec{w}$ w.r.t. the basis $\mathcal{B}$.

How do we know $\mathcal{B}$ is a basis?