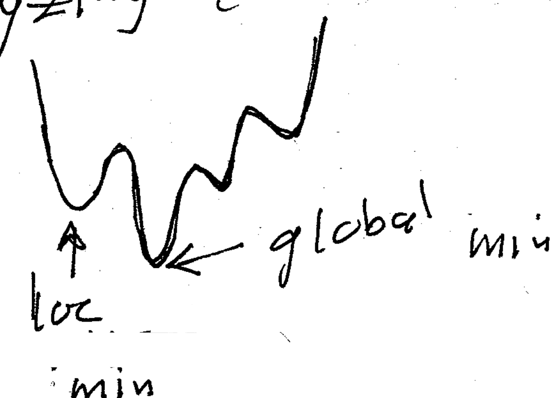
LADS 129

1

Optimization $\rightarrow$ analyzing cost function

$\Phi : \mathbb{R}^n \rightarrow \mathbb{R}$



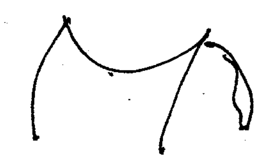
<u>2nd derivative test</u> $\Phi : \mathbb{R}^n \to \mathbb{R}$ C^2

and $\vec{x}_0$ is a <u>critical point</u> $\nabla \Phi(x_0) = \vec{0}$

Let $\{\lambda_1, \ldots, \lambda_n\}$ be the spectrum of $H\Phi(x_0)$ (2)

$\Rightarrow$ (1) all $\lambda_i > 0 \Rightarrow$ loc min 

(2) $\lambda_i < 0$ " MAX 

(3) Some $\lambda_i < 0$ some $\lambda_i > 0$ 

$\Rightarrow$ Saddle

(4) Some $\lambda_i = 0 \Rightarrow$ no test.

Process

(1) Compute $\nabla \Phi$, $H\Phi$

(2) Find all crit pts $\nabla \Phi(\vec{x}) = 0$

(3) Find the spectrum of $H\Phi(x_0)$ at each crit pt x_0.

(4) Classify the critical points using the 2nd Derivative Test

4

$$\Phi(x_1,x_2) = x_1^4 + x_2^4 - 4x_1x_2 + 1$$

$$\nabla\Phi(x_1,x_2) = \left[4x_1^3 - 4x_2,\ 4x_2^3 - 4x_1\right] = \vec{0}$$

$$4x_1^3 - 4x_2 = 0 \rightarrow x_2 = x_1^3$$

$$4x_2^3 - 4x_1 = 0$$

$$x_1(x_1^8 - 1) = x_1^9 - x_1 = 0$$

$$x_1 = 0 \Rightarrow x_2 = 0$$

$$x_1 = \pm 1 \qquad x_2 = \pm 1$$

$(0,0)$, $(1,1)$, $(-1,-1)$ CRIT pts.

(5)

$$H\Phi(x_1, x_2) = \begin{bmatrix} 12x_1^2 & -4 \\ -4 & 12x_2^2 \end{bmatrix}$$

(1) $(0,0)$, $H\Phi(0,0) = \begin{bmatrix} 0 & -4 \\ -4 & 0 \end{bmatrix}$

$$\begin{vmatrix} -\lambda & -4 \\ -4 & -\lambda \end{vmatrix} = \lambda^2 - 16, \quad \lambda = \pm 4 \Rightarrow \text{Saddle point.}$$

(2) $(1,1)$, $H\Phi(1,1) = \begin{bmatrix} 12 & -4 \\ -4 & 12 \end{bmatrix}$

$$\begin{vmatrix} 12-\lambda & -4 \\ -4 & 12-\lambda \end{vmatrix} = (12-\lambda)^2 - 16 = \lambda^2 - 24\lambda + 128$$

$$= (\lambda - 8)(\lambda - 16) \quad \lambda = 8, 16$$

$\Rightarrow$ loc. min.

(3) $(-1,-1)$, $H\Phi(-1,-1) = \begin{bmatrix} 12 & -4 \\ -4 & 12 \end{bmatrix}$

so again $\lambda = 8, 16$ so loc min