Procedure for computing SVD by hand when A has linearly independent columns

(1) Form A^TA which is symmetric, positive definite

(2) Find the eigenvalues $(\lambda_1, .., \lambda_k)$ and eigen vectors $\vec{w}_1, .., \vec{w}_k$ of A^TA

(3) The singular values are $\sigma_i = \sqrt{\lambda_i}$ and the right singular vectors are $\vec{v}_i = \frac{\vec{w}_i}{\|w_i\|}$

(4) $V = \left[\vec{v}_1\, \vec{v}_2 \,..\, \vec{v}_k\right]$ is automatically orthogonal

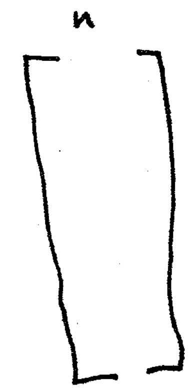
(5) The left singular vectors are $\vec{u}_i = \frac{A\vec{v}_i}{\sigma_i}$ (note $\sigma_i > 0$ since A^TA is pos def)

(6) $U = \left[\vec{u}_1 \cdots \vec{u}_j\right]$ and $\Sigma = \text{diag}(\sigma_1, .., \sigma_\ell)$

NOTE This works for nonsquare A as long as its columns are linearly independent yields reduced SVD

Example

- A is $m \times n$ $\quad m > n$ with lin ind col



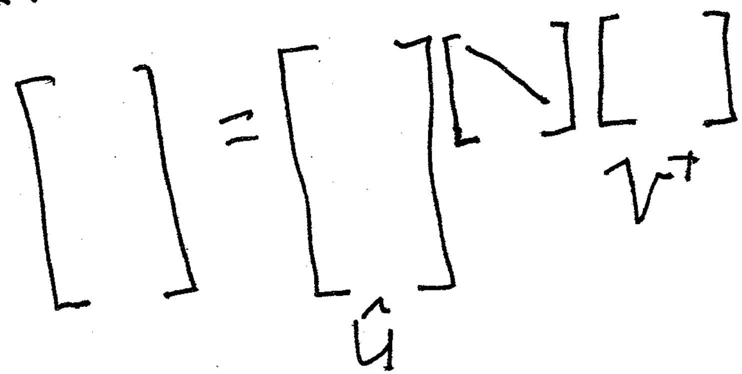
- $A^T A$ is $(n \times m) \cdot (m \times n) = n \times n$

  So there are n λ_i and thus n σ_i

- Thus Σ is $n \times n$ and V^T is $n \times n$

- But $\dfrac{A\vec{v}_i}{\sigma_i}$ will be a $m \times 1$ vector so

  $\hat{U}$ will be $m \times n$



we write $\hat{U}$ since it is not full $n \times n$

- This is reduced or Thin SVD and will be discussed in the next lecture.