Although the step function \( f(x) = 0 \) for \( x \leq 0 \), \( f(x) = 1 \) for \( x > 0 \)

has \( f(0) = 0 \), you don't want to construct decision lines or sets where the input to \( f \)

is exactly zero. This would be unstable to small perturbations and thus not robust.

So in the example on pages 7 and 8 of Lecture 27, the line \( x_1 - x_2 = 0 \) does yield \( f(\mathbf{0}) = 0 \)
correctly, we don't want to use it.

Rather, the decision line should avoid the data input points

\( x_1 - x_2 + y_2 = 0 \)
This instability is one reason the ramp or sigmoid are preferred in general. They are continuous so small changes in input yield small changes in output, rather than a big jump like \( \Delta s \) has at zero.
We now see what a single neuron with ReLU step activation does

\[ x_1 \xrightarrow{w_1} (\circ) \rightarrow \text{output} = \begin{cases} 0 & w_1 x_1 + w_2 x_2 + b \leq 0 \\ 1 & w_1 x_1 + w_2 x_2 + b > 0 \end{cases} \]

So \( F(x) = \sum_s (w^T x + b) = \sum_s (w_1 x_1 + w_2 x_2 + b) \)

Now \( 0 = w_1 x_1 + w_2 x_2 + b \) is a line in the \((x_1, x_2)\)-plane.

It divides the plane into two halves:

- One where \( F(x) = 0 \)
- One where \( F(x) = 1 \)
Example: Find the weights and bias of a single neuron with a step-activation that classifies the points

\((-1,-1), (-1,0), (0,1)\) → have value 1 (yes)

\((0,0), (1,0), (1,1)\) → have value 0 (no)
Solve Plot Then in the plane

\[ \text{value} = 1 \]

\[ x \]

\[ \times \]

\[ x \]

\[ \times \]

\[ \circ \text{ value} = 0 \]

We find a "decision line" that divides the

one such line is $x_1 - x_2 + \frac{1}{2} = 0$

but $F(x_1, x_2) = \sqrt{(x_1 - x_2 + \frac{1}{2})}$

yields $F(0, 0) = \sqrt{(\frac{1}{2})} = 1$

The wrong value so we use the description $-x_1 + x_2 - \frac{1}{2} = 0$

So the soln is $w_1 = -1, w_2 = 1, b = -\frac{1}{2}$