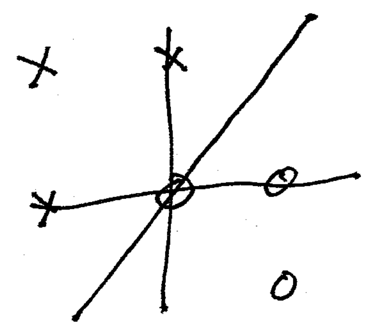
Although the step function $\sigma_s(x) = 0 \quad x \le 0$
$\qquad\qquad = 1 \quad x > 0$

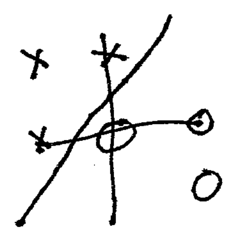
has $\sigma_s(0) = 0$, you don't want to construct decision lines or nets where the input to σ is exactly zero. This would be unstable to small perturbations and thus not robust.

So in the example on pages 7 and 8 of Lecture 27



the line $x_1 - x_2 = 0$ does yield $\sigma(0,0) = 0$ correctly, we don't want to use it

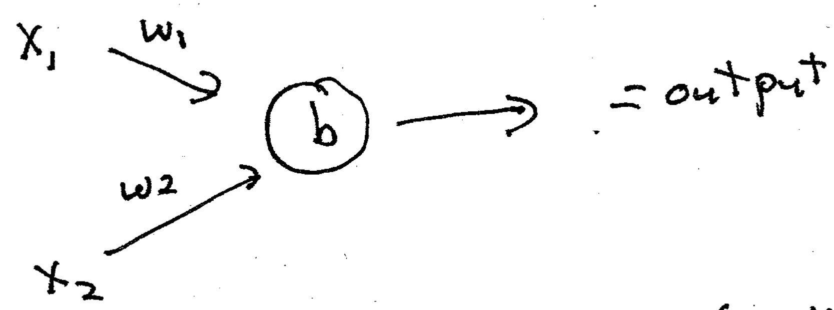
rather, the decision line should avoid the data input points



$x_1 - x_2 + 1/2 = 0$

This instability is one reason the ramp or sigmoid are preferred in general. They are continuous so small changes in input yield small changes in output, rather than a big jump like σ_s has at zero.

We now see what a single neuron with the step activation does



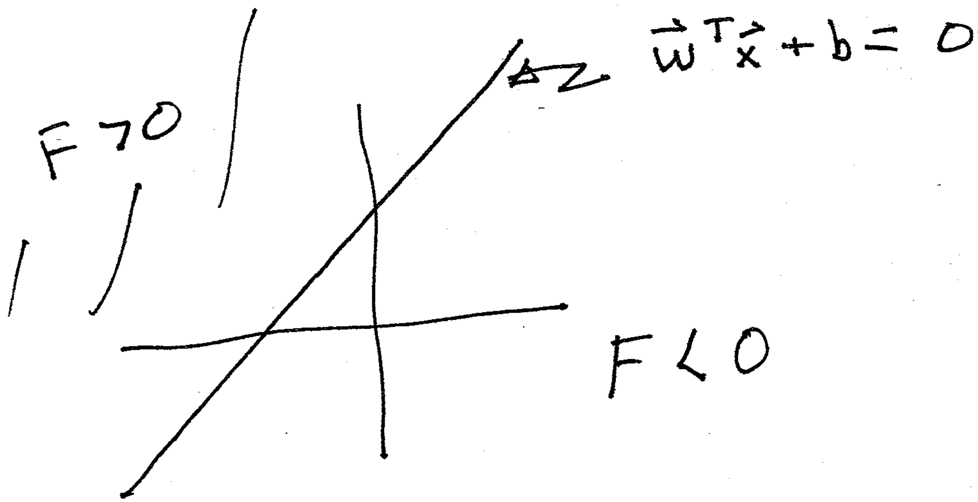
So $F(\vec{x}) = \sigma_S(w^T \vec{x} + b) = \sigma_S(w_1 x_1 + w_2 x_2 + b)$

$$= \begin{cases} 0 & \text{when } w_1 x_1 + w_2 x_2 + b \leq 0 \\ 1 & w_1 x_1 + w_2 x_2 + b > 0 \end{cases}$$

Now $0 = w_1 x_1 + w_2 x_2 + b$ is a line in the (x_1, x_2)-plane

It divides the plane into two halves

one when $F(\vec{x}) = 0$ and one where $F(\vec{x}) = 1$

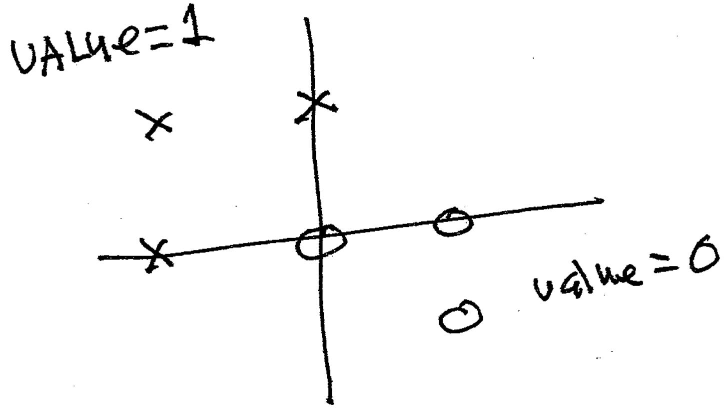


<u>Example</u>: Find the weights and bias of a single neuron with a step-activation that classifies the points

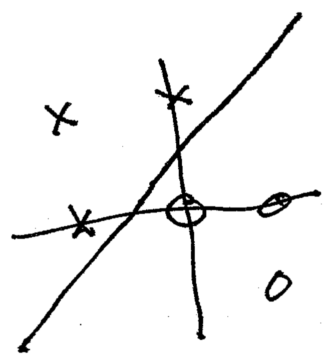
$(-1,-1), (-1,0), (0,1) \longrightarrow$ have value 1 (yes)

$(0,0), (1,0), (1,-1) \longrightarrow$ have value 0 (no)

<u>Soln</u> Plot them in the plane



We find a "decision line" that divides them



one such line is

$$x_1 - x_2 + 1/2 = 0$$

but $F(x_1, x_2) = \sigma(x_1 - x_2 + 1/2)$

yields $F(0,0) = \sigma(1/2) = 1$

the wrong value so

we use the description $-x_1 + x_2 - 1/2 = 0$

So the soln is $w_1 = -1, w_2 = 1, b = -1/2$