Read all these instructions before starting. After you have read them, turn the page and start timing your exam.

1. You have 1.5 hours to do the exam. This does not include the time that it might take you to scan your soln and upload them. You may also write your solutions directly on a tablet.
2. Exams must be uploaded to Assignments by 5:00 PM, Thursday, October 21 (FL time)
3. The exam is open book and notes, but only those for this course.
4. Set aside 1.5 hours where you can work undisturbed. During this time you cannot talk to anyone or consult any web resources other than those for this course.
5. The questions sheets are a list of the questions and there is no place to put your answers, so write your answers on separate pieces of paper. You can print the question sheet, or look at it on-screen, but do not include them in your scanned soln.
6. Keep in mind that I have to grade these, so please write neatly, organize your answers, and have your solutions in the same order as the questions.
7. Give complete answers. In your proofs you may use any result that we have proved in class or any homework or review problem.
8. You are bound to all the stated conditions for the exam by the UF Honor code.
9. Your scanned solutions must contain the statement:

   "I have spent at most 1.5 hours working the exam and while I was taking the exam I have consulted no one nor nor any book nor any web resources other than the web pages for this course"

   Then your signature and UF ID number.

TOPOLOGY EXAM 1 • FALL 2021 • PROF. BOYLAND

1.(10 *points*) Let Y be all finite subsets of $\mathbb{Z}_+$. Show that Y is countable.

2.(20 *points*) Prove or disprove.

(a) $\overline{A \times B} = \overline{A} \times \overline{B}$

(b) $\mathrm{Int}(A \cap B) = \mathrm{Int}\, A \cap \mathrm{Int}\, B$

3.(10 *points*) Recall that in the countable complement topology on X a set is open exactly when its complement is countable or is all of X. Give $\mathbb{R}$ the countable complement topology.

(a) To which point or points does the sequence $x_n = 1/n$ converge?

(b) What is $\overline{\{2\}}$?

4.(20 *points*) For a list of real numbers a_i and b_i define a map $f : \mathbb{R}^{\mathbb{Z}_+} \to \mathbb{R}^{\mathbb{Z}_+}$ by

$$f(x_1, x_2, \dots) = (a_1 x_1 + b_1, a_2 x_2 + b_2, \dots).$$

(a) What conditions if any on the a_i and b_i are necessary so that f is continuous when the domain and range are both given the box topology.

(b) What conditions if any on the a_i and b_i are necessary so that f is continuous when the domain and range are both given the uniform topology.

Do 3 (three) and only 3 of the following 4 problems

5.(10 *points*) If $(Y, \mathcal{T}_Y)$ is a topological space and $f : X \to Y$ is a surjective function, is

$$\{f^{-1}(U) \colon U \in \mathcal{T}_Y\}$$

a topology on X? Prove your answer.

6.(10 *points*) If A is dense in B and B is dense in C, is A dense in C? Prove your answer.

7.(10 *points*) Given $(X, \mathcal{T}_X)$ and $(Y, \mathcal{T}_Y)$ show that $(X \times Y, \mathcal{T}_{X \times Y})$ is Hausdorff if and only if $(X, \mathcal{T}_X)$ and $(Y, \mathcal{T}_Y)$ are.

8.(10 *points*) Assume that d and d' are metrics on X both with $\mathrm{diam}(X) = \sup\{d(x, y) \colon x, y \in X\} \leq 1$ and

$$(d(x, y))^2 \leq d'(x, y) \leq \sqrt{d(x, y)}$$

for all $x, y \in X$. Show that d and d' induce the same topology.

Don't forget your signed statement