

Subgroups

Abelian Groups

LAST time: $\sum G_\alpha$ $\forall \alpha \in I$, $G_\alpha \subseteq G$

We defined $\bigoplus_{\alpha \in I} G_\alpha$ called "internal direct sum".

Extending! Assume G is given with $\forall \alpha \in I$

Monomorphism $G_\alpha \hookrightarrow G$

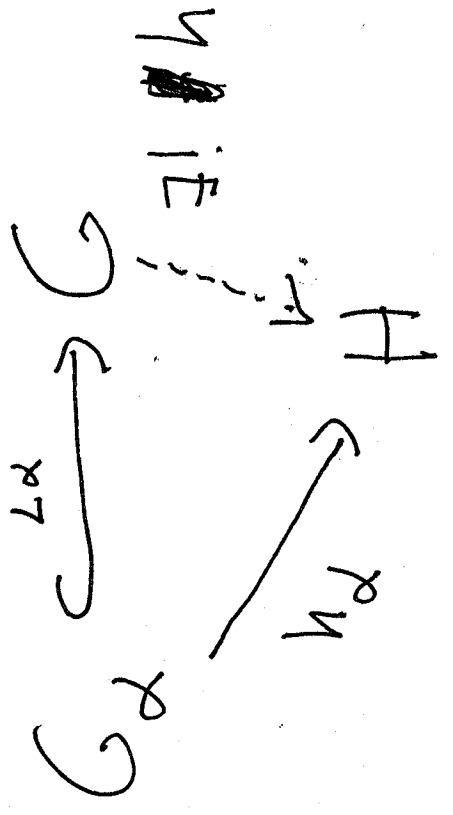
is the external direct sum $G = \bigoplus_{\alpha \in I} G_\alpha$

or a little sloppy write ~~G~~ G and H is a

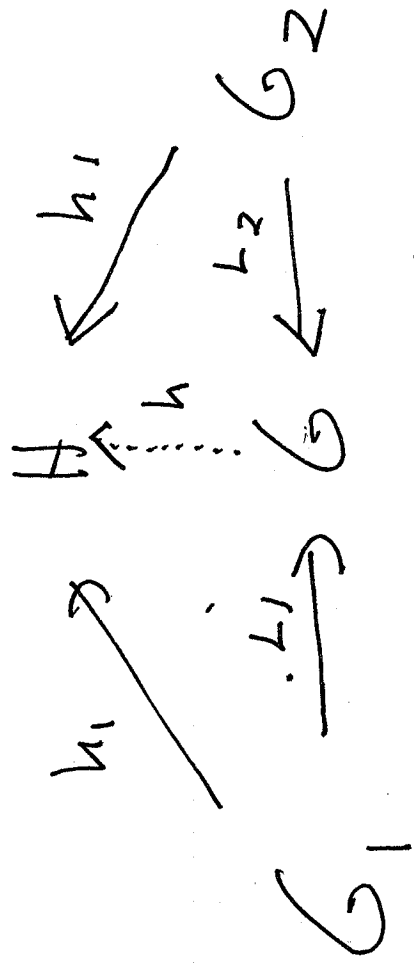
$$G = \bigoplus G_\alpha$$

Lemma Assume $G = \bigoplus_{\text{ISX}} G_\alpha \Rightarrow$

Given any abelian group and homomorphism $f: H \rightarrow G$ for some abelian group H $f = \sum f_\alpha$ where $f_\alpha = f \circ i_\alpha$ and $i_\alpha: H \rightarrow G_\alpha$ is the inclusion map.



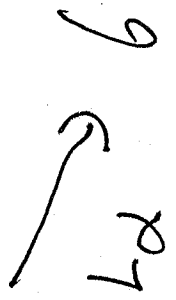
eg: $\alpha \in \mathbb{Z}, \mathbb{Z}$



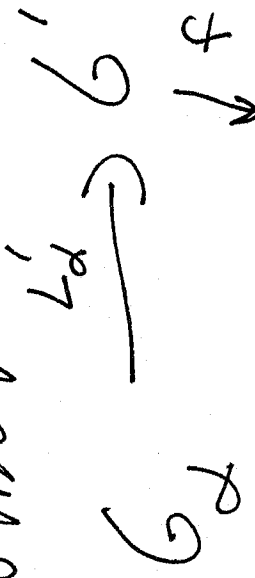
CO product in the category of Abelian groups

groups

Prop: These properties ~~are~~ characterize the direct ~~sum~~ sum. Specifically,

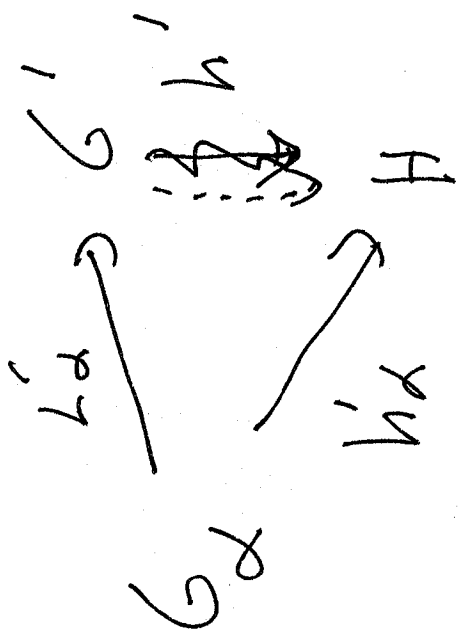


isomorphism $\rightarrow f \downarrow$



with $f \in F$

fields G, G at isom. s.t. $G \cong G$

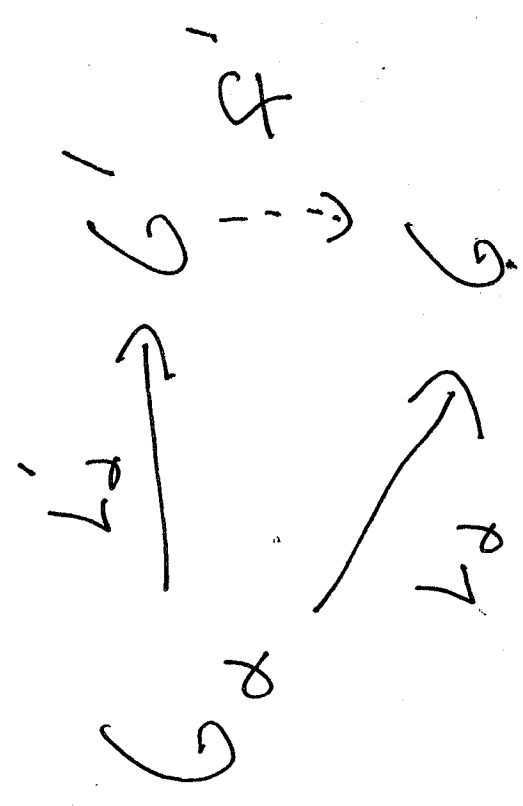


$\forall f \in G \subseteq H \in G; r_1$ sums

maps $H \neq G \rightarrow G; r_1$

Let G be any abelian with

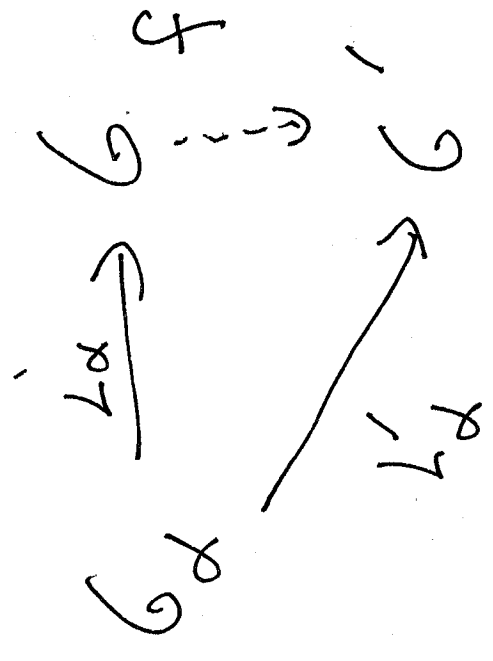
Proof! To start, Treat G as H and L_α to be h'_α or



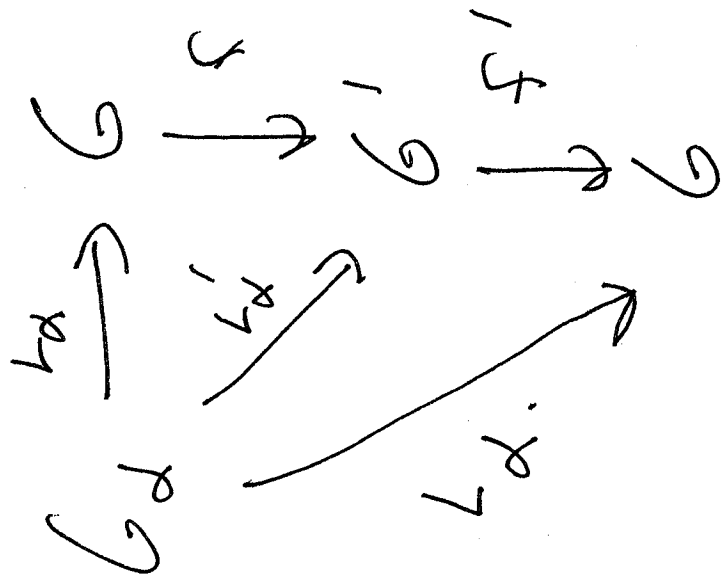
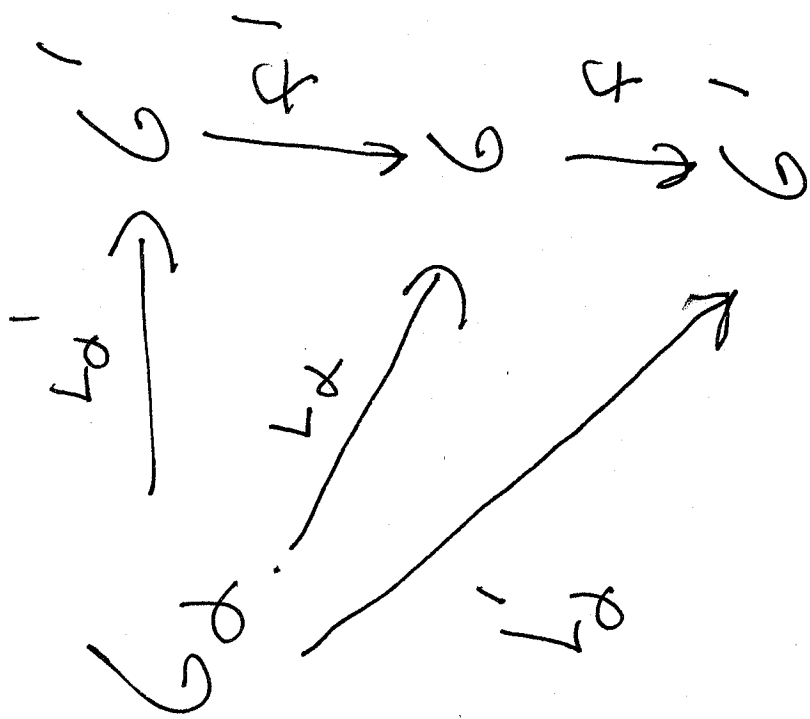
Assume G has the started property

$\Rightarrow F$

switch roles



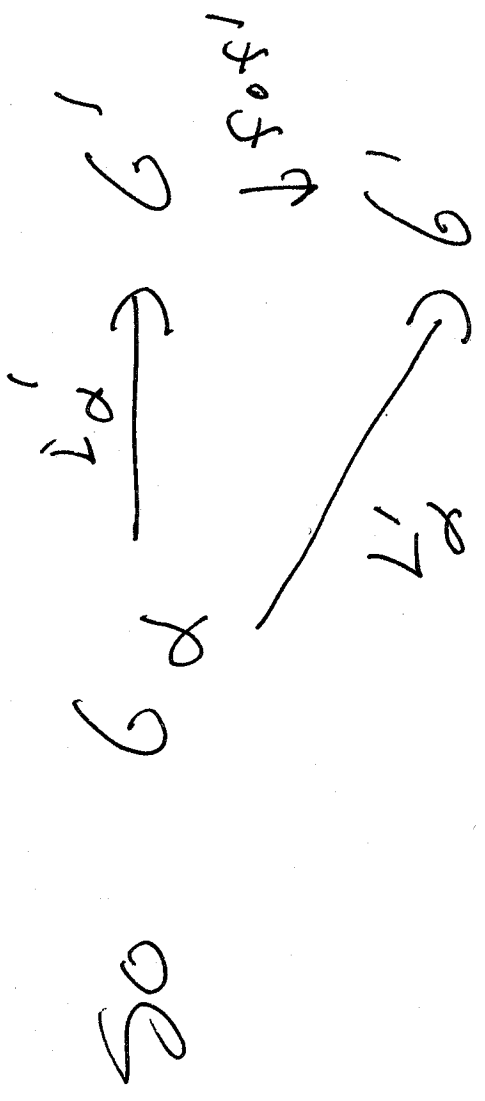
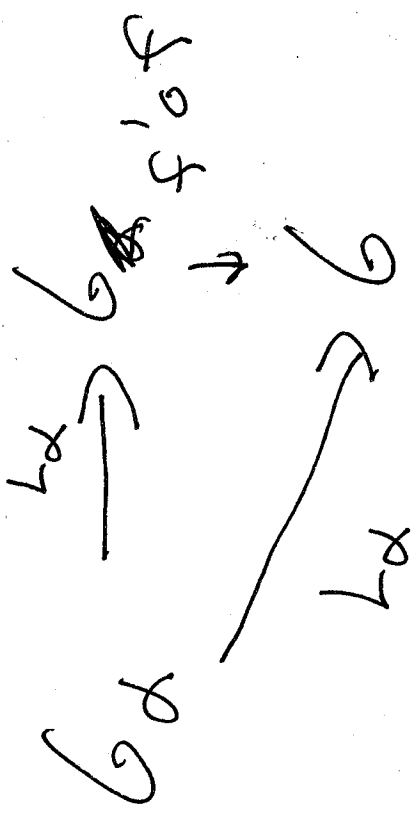
Stack be diagrams



~~Both~~



most sp as 150M.
 if prof $\in \mathbb{R}$ $p_1 = 20,5$ $p_1 = 150,5$
 By uniqueness for nof p_7 p_7 p_7 p_7
 These also commute with p_1 p_7 p_7 p_7
 and p_7 p_7 p_7 p_7

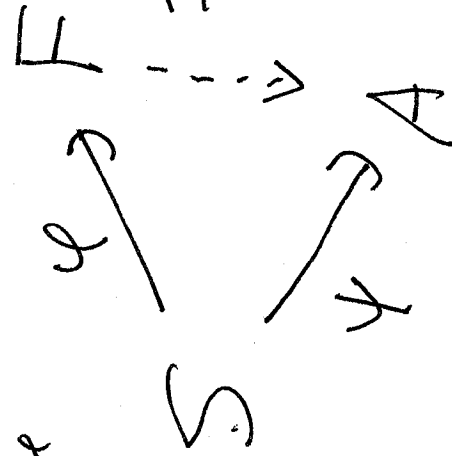


Free Abelian Groups

Diagram Definition: S is a set. A is a group. F

Free Abelian group on $S \rightarrow F$ so that

with a \forall injective

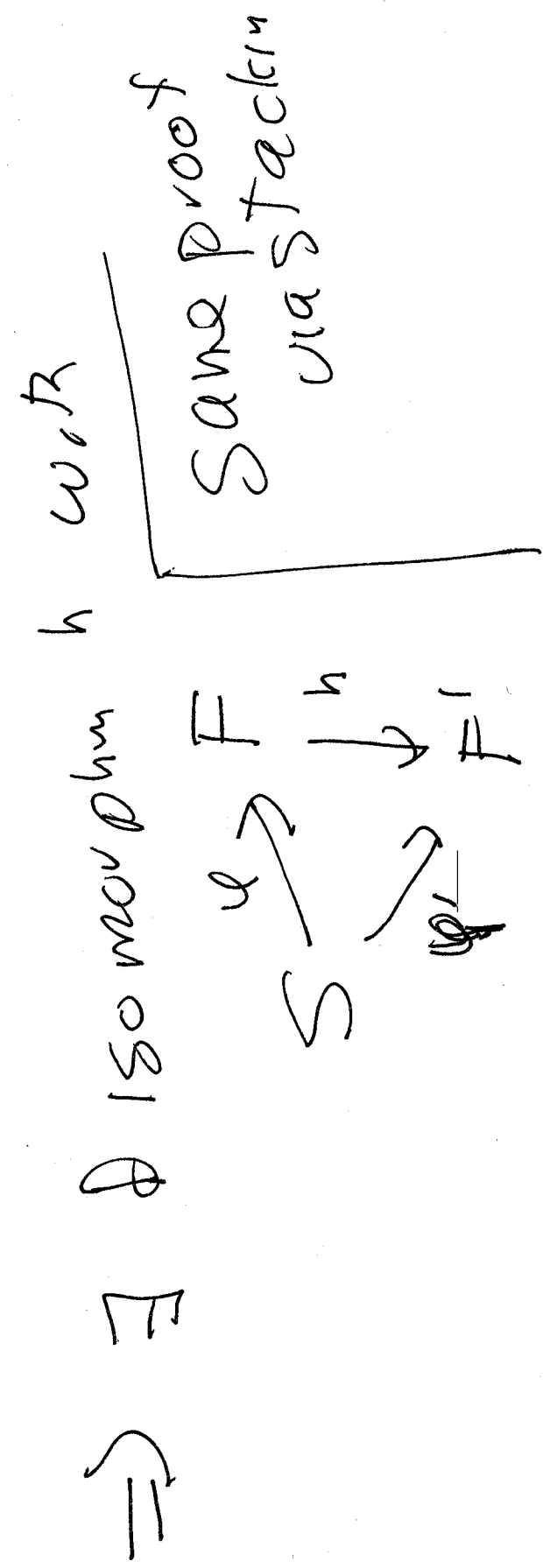
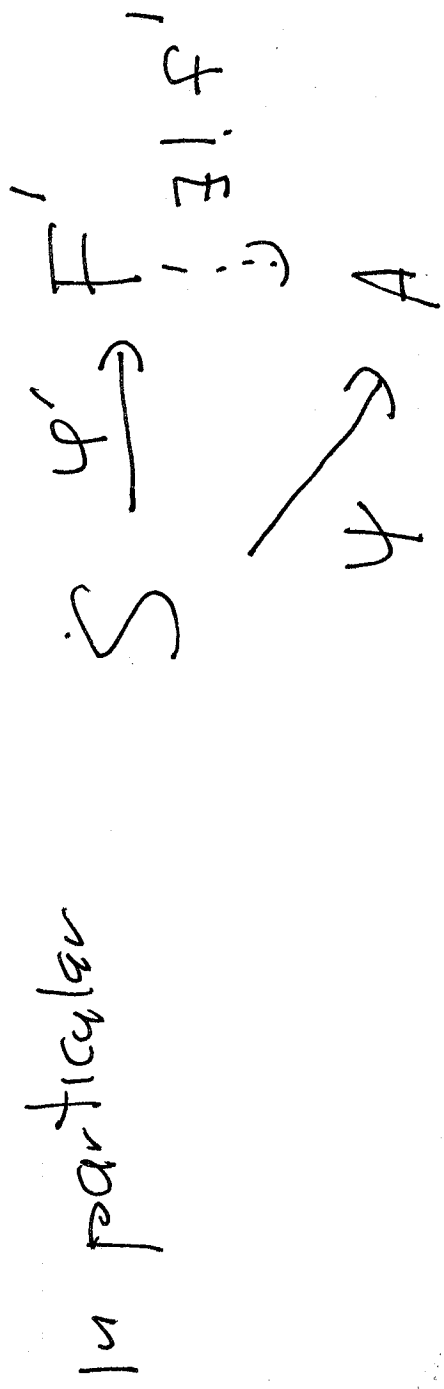


$\exists!$ is a homomorphism

Any abelian group A any set function ψ .

Thm 10.1 This characterizes the

Free group w.r.t. to \mathcal{U} and \mathcal{S} .



Concrete definition!

Let F be all formal finite

sums:

$$\sum_{i=1}^k n_i x_{d_i}$$

$$n_i \in \mathbb{Z}$$

with all d_i distinct, $x_{d_i} \in S$

with the obvious operation.

Thm: This concrete realization
 \Rightarrow satisfies the diagrammatic def.

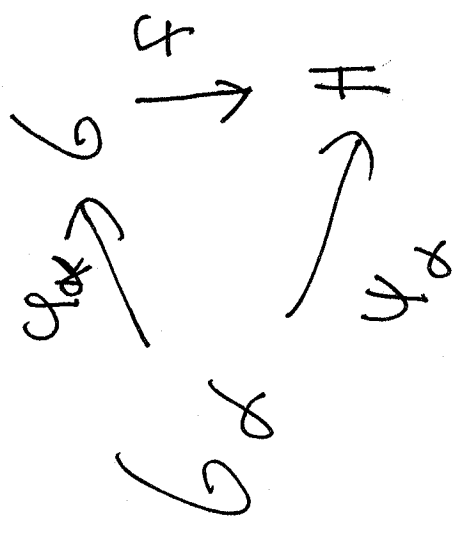
General Case

Free product

$\{G_\alpha\}_{\alpha \in I}$, $\varphi_k: G_\alpha \rightarrow G$ mono.

G is the free product

and $\varphi_\alpha: G_\alpha \rightarrow H$ mono \Rightarrow



This characterizes de Free product.

Concrete realization - All words

$x_{d_1} x_{d_2} \dots x_{d_k}$

adjacent x_{d_L} from different G_α

with obvious operation - concatenation
and reduce.

Thm: This satisfies the diagrammatic def.

Example

generators and relations

$$G_1 = \langle \{a\} \mid \{a^3\} \rangle$$

$$G_2 = \langle \{b\} \mid \{b^5\} \rangle$$

$$G_1 * G_2 = \langle \{a, b\} \mid \{a^3, b^5\} \rangle$$

Free product

$$G_1 = \langle a \mid a^3 \rangle = \mathbb{Z}_3$$

$$G_2 = \langle b \mid b^5 \rangle = \mathbb{Z}_5$$

$$\mathbb{Z}_3 * \mathbb{Z}_5 = \langle a, b \mid a^3, b^5 \rangle$$

$$a^3 = e = b^5$$

direct free product

$$\mathbb{Z}_3 \oplus \mathbb{Z}_5$$

NOTATION

$G_1 * G_2$

$\prod_{\alpha \in I} G_\alpha$

or

$\ast_{\alpha \in I} G_\alpha$

Free groups - diagrammatic - Same as
free Abelian but now $A = H$ is g

general group.

$X_{\alpha_1}^{n_1} \dots X_{\alpha_k}^{n_k}$

All words

Concrete

$n_k \in \mathbb{Z}$

concatenation with cancellations.

With free group - ~~$F_{a,b}$~~

$$F_1 = \langle a \rangle$$

$$F_2 = \langle b \rangle$$

$$F_1 * F_2 = \langle a, b \rangle = F_2$$

~~✓~~

$$a^{n_1} b^{n_2} a^{n_1} b^{n_2}$$