DEF \quad A \subseteq (X, \mathcal{Y}) \quad x \text{ is a limit point of } A, \text{ if } U \text{ is a nbhd of } x

\Rightarrow \quad U \cap (A - \{x\}) \neq \emptyset

or \quad x \in cl(A - \{x\})

Example \quad \exists \frac{1}{n}. n \in \mathbb{Z}^+ = K

Only \ limit \ point \ is \ \overline{x_0}.

\overline{K} = K \cup \overline{x_0}

Theorem \quad A = A \cup A' \quad (A' = \text{all limit points of } A)
(2) $A \subseteq \overline{A}$ by def.

So assume $x \in A$.

⇒ by def any $u$ and $d \vdash x$ hits

$A - \exists x \exists \Rightarrow x \in \overline{A}$ by a previous $m\in m$

so $A' \subseteq \overline{A}$.

(⇒) $x \in \overline{A}$ $x \in A$ done.

So assume $x \notin A$. By a $m\in m$ every

$u$ and $d \vdash x$ has $u \land A \neq \emptyset$, but $x \notin A$

so $u \land \exists x \exists \gamma \neq \emptyset \Rightarrow x \in A$. 
NHA is finite.

and some infinite set is such that

Way of contradiction: assume \( \exists A \).

**Proof:** \( \Rightarrow \) Recall \( \Rightarrow \) BUC \( \Rightarrow \)

Contrain infinite many points \( A \).

Prove \( \Rightarrow \) all levels \( x \)

(Finite sets are closed). Then

**Theorem:** \( (x, y) \) is \( \Delta \)
\[ U \cap A \text{ is finite} \]

Let \( U \cap (A \cap \mathbb{R}^3) = \{ x_1, \ldots, x_7 \} \).

By Theorem 5, \( U - \{ x_1, \ldots, x_7 \} \) open.

Thus \( U \cap (\mathbb{R}^3 - \{ x_1, \ldots, x_7 \}) \) is a neighborhood \( x \) that misses \( A - \mathbb{R}^3 \).

\( x \) is not a limit point.
Sequences

$$\exists x_n^3, n \in \mathbb{Z}^+$$

Formally is a map $\phi: \mathbb{Z}^+ \to X$

and we write $x_n = \phi(n)$

or $\exists x_n^3$ is countable collection

of points in $X$.

\[
x_n \to x \quad \text{or} \quad \lim_{n \to \infty} x_n = x \quad \text{or} \quad \{x_n\} \text{ converges } x \text{ in } \mathcal{F}.
\]
For all \( u,d, x, \in \mathbb{N} \) so that \( u \geq N \Rightarrow x_n \in U \).

i.e. all neighborhoods of \( U \) contain all but finitely many points from the sequence.

\[
\text{calculus in } \mathcal{TR}_5
\]

given \( u \leq 70 \), \( U = \mathbb{N} \setminus (3+3 \times (x-x-n)) \)

\[
\Rightarrow \exists N, u \geq N \Rightarrow x_n \in U
\]

\[
= \|x - x_n\| > \epsilon
\]
(1) \[ x_n = \frac{1}{n}, \quad \frac{1}{n} \to 0 \]

(2) \[ x_n = b \text{ for all } n. \]

\[ x_n \to b \]

Every nhbd of a contains b. \( x_n \to a \) also and \( x_n \to c \) also.

(3) \[ x_n = \frac{3}{15^n} \text{ doesn't converge.} \]
Theorem $(X, Y)$ is HD $\implies$ a sequence converges to at most one point.

Proof Assume $x_n \to x$ and pick $y \neq x$. Since $Y$ is HD, there exist open $U, V$ such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$.

Since $x_n \to x$, $U$ contains all but finitely elements of $\{x_n\}$.

So $x_n \not\to y$.

Doesn't converge.
Continuous Functions

**DEF:** \( f: (X, \mathcal{J}_X) \rightarrow (Y, \mathcal{J}_Y) \) is continuous if for every \( V \in \mathcal{J}_Y \)

\[ f^{-1}(V) \in \mathcal{J}_X \]

"Inverse images of open sets are open."

**Note:** (1) depends on \( f \) and the topology on the range and domain.

(2) If \( B \) is a base for \( \mathcal{J}_Y \) suffices to have \( f^{-1}(B) \in \mathcal{J}_X \) \( \forall B \in B \).
(3) If $A$ is a subbase for $\mathcal{J}_y$ \\
$\Rightarrow$ suffices to check $f^{-1}(S) \in \mathcal{J}_x$ \\
for all $S \in A$.

Pictures of graphs for $f: \mathbb{R}_x \rightarrow \mathbb{R}_y$

$\begin{align*}
&f^{-1}(U) \\
&f(U) \\
&f(U) \text{ not open.}
\end{align*}$
Connecting to Calculus def for \( f : \mathbb{R} \to \mathbb{R} \)

\[
\text{Calc DEF}: \forall x_0, \forall \varepsilon > 0 \exists \delta > 0 \text{ s.t. } \forall x, \|x - x_0\| < \delta \implies \|f(x) - f(x_0)\| < \varepsilon.
\]

\[
\text{Top DEF } \implies \text{Calc DEF}
\]

<table>
<thead>
<tr>
<th>Given ( x_0, \varepsilon &gt; 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>form ( T = (f(x_0) - \varepsilon, f(x_0) + \varepsilon) ) open in ( \mathbb{R} )</td>
</tr>
</tbody>
</table>

\( \text{Calc DEF}: \forall x_0, \forall \varepsilon > 0 \exists \delta > 0 \text{ s.t. } \forall x, \|x - x_0\| < \delta \implies \|f(x) - f(x_0)\| < \varepsilon. \)
By Theorem 1, $f^{-1}(A)$ is open.
Example

\[ \text{id: } \mathbb{R} \to \mathbb{R} \]

Check base elements

\[ (\text{id})^{-1}(a, b) = (a, b) \text{ open } \mathbb{R} \]

So cont

\[ \text{id: } \mathbb{R} \to \mathbb{R} \]

Check base elements \((9, 55), (59, 6)\)

\[ (\text{id})^{-1}(59, 6) = \{59, 6\} \text{ not open in } \mathbb{R} \]

So not cont.
In general,

$Y_1 \supseteq Y_2 \text{ on } X \iff$

$\text{id}: (X, Y_1) \to (X, Y_2) \text{ is continuous.}$