Reminder Def. \((X, \mathcal{O})\)

1. Separation:
   a. \(X = C \cup D\)
   b. \(C \cap D = \emptyset\)
   c. \(C, D \neq \emptyset\)
   d. \(C, D\) open. In \(\overline{X}\)
   e. no separators. \(\Rightarrow\) connected

2. \(\alpha: \Sigma_{011} \rightarrow X\) \(\alpha(0) = x\) \(\alpha(1) = y\)
   is a path \(x \leftrightarrow y\).

3. \(\forall x, y \in X\) \(\exists\) path \(x \leftrightarrow y\)
   \(\Rightarrow X\) is path connected.
Main results and rays

1) Intervals in \( \mathbb{R} \) are connected.

2) P.C. \( \Rightarrow \) C.

3) Continuous image of C. \( \Rightarrow \) is C.

Connected not path connected.
**Parts d spaces in \((X,Y)\).**

1. **Def.** \(x \approx y \iff \exists \text{ connected } C \subseteq X \) with \(x, y \in C\). \[\]

This is an *equivalence relation*.

- reflexive: \(x \approx x\)
- symmetric: \(x \approx y \Rightarrow y \approx x\)
- transitive: \(x \approx y, y \approx z \Rightarrow x \approx z\)

\[\]

\(y \in C_1 \cap C_2, C_1, C_2 \text{ conn} \Rightarrow C_1 \cup C_2 \text{ is conn. and contains } x \text{ and } y.\)
Equivalence classes of \( x \) are called connected components or just components.

\[ \rightarrow \rightarrow \rightarrow \]

To prove, connected components are connected.

Path comp are path conn.

**Fact:** \( x \rightarrow y \Rightarrow x \cup y \)

So connected components are unions of path components.
1 component
2 path components.

path connected
Example

\[ x = 0.15 \nu \]

Each pt is a component.
Proof (1) follows from equivalence relation.

(2) Fix \( x \) and \( x_0 \in C_x \)

\[ \forall y \exists A_y \text{ connected with } \]

\[ x, y \in A_y \land x \in \bigcup A_y. \]

Each \( A_y \) connected \( \Rightarrow C_x = \bigcup A_y \) is connected.

(3) Since each \( C_x \) is connected.
If \( \exists \chi \) are the path components of \( X \)

(1) \( \bar{X} = \bigcup_{x \in \chi} x \) (\( \bigcup \) means disjoint union)

(2) Each \( C_x \) is path connected.

(3) \( A \subset X \) is path connected

\[ \Rightarrow \exists ! x \text{ with } A \subset C_x \]

exists unique.
DEF: $X$ is locally connected at $x$ if for all open $U$ containing $x$,

$$T_{\text{open and connected with}}$$

$$x \in U \subseteq U$$

If locally connected at all points,

$$\Rightarrow X \text{ is locally connected.}$$
1. Intervals and Rays in $\mathbb{R}$ are locally conn and loc path conn.

2. $[0, 1) \cup (1, 2]$ not conn.
   loc path conn
   loc conn

3. 
$X = \{ 0 \leq \Sigma_{ij} x_{i} x_{j} \leq 1 \} \times \mathbb{R}^{2}$

$Y = \text{all arcs connecting points on } \partial X$

to the point $(\frac{1}{2}, 1)$

loc path con

not loc con