easy: this is topology.

Let \( T \) = all \( T \) -regular unions of \( T \) -regular sets.

easy: this is a base.

Let \( \mathcal{B} = \{ T \mid T \supseteq \text{ finite intersections of } \mathcal{B} \} \). 

Example: \( \mathcal{B} = \{ \emptyset, \{ a \}, \{ a, b \} \} \). 

So \( \mathcal{S} = \mathcal{S} \cup \{ a \} \cup \{ a, b \} \) is a subbase. 

For \( \mathcal{S} \) is a subbase if 

\( R \subseteq \mathcal{S} \). 

For \( \mathcal{S} \) is a subbase if 

}\)
Back to example

\[ S = \frac{1}{2} \sum_{n=1}^{\infty} (q_{1,0})^{n}, \quad q_{1,0} > 0 \quad \alpha \in \mathbb{R}^{3} \]
Order topology on strictly linear ordered sets.

Let $B$ be all sets of the form:

1. $(a, b)$
2. $[a_0, b)$ if $a_0$ is the smallest element if it exists.
3. $(a, b_0]$ if $b_0$ is the largest element if it exists.

**Fact:** This is a base that generates the order topology.
Examples

(1) \((\mathbb{R}, \leq)\) yields standard topology with base \(\mathcal{B} = \{(a, b) : a < b\}\)

(2) Dictionary order on \(\mathbb{R}^2\)

\(\leq ((a, b), (c, d))\) with \(a < c\)

\((a, b), (a, c)\) \(b < c\)

not the standard topology.
\( (\mathbb{Z})_+ = \ldots 3, 3, 3, 3 \)

has smallest element 1.

\[ \sum_{1, 2} = 3 \cdot 3 \]

\[ (n-1, n+1) = 3 \cdot n \cdot \frac{3}{3} \]

discrete topology.
(4) \( \mathbb{Z} \times \mathbb{Z} \) with the dictionary topology.

\[ (1, 1) \text{ smallest } \]

\[ \text{every pt is open but } (2, 1) \]

any open interval containing \((2, 1)\) contains a point \( < (2, 1) \) or \((4, b) \) for some \( b \)

and thus \((1, b)\) for all \( b \geq 6 \)

so infinitely points.
Given \((X, T_X)\) and \((Y, T_Y)\) and
\[ T_{X \times Y} (y, y) \Rightarrow \text{de topology on} \]
the product \(X \times Y\) is \(\bigotimes_{x \times y} \) which
is generated by the base
\[ \bigcup \{ U \times V \mid u \in T_X, v \in T_Y \} \]
Check B is a base.

\[ (a, b) \times (c, d) \text{ is typical base element.} \]

\[ \{ (u_1, v_1) \in (u_2, v_2) \times (u_3, v_3) \mid (u_1, v_1) \in (u_2, v_2) \times (u_3, v_3) \} \]

\[ \{ (u_1, v_1) \times (u_2, v_2) \} \]

Theorem 1: Intersection of base elements yields a base element.
B is not a topology i.e. the only open sets in $\mathcal{J}_{\mathbb{x,y}}$ are not just $U \times V$

eq \forall \left((U_1 \times V_1) \cup (U_2 \times V_2)\right) \in \mathcal{J}_{\mathbb{x,y}}

but is not a product
Now reduce to bases in \( X \) and \( Y \).

**Theorem**: If \( B \) is a base for \( J_X \) and \( C \) is a base for \( J_Y \) then:

\[
B = \{ B \times C \mid B \in B, C \in C \}
\]

is a base for \( J_{X \times Y} \).

**Uses Lemma**: \( F \) is a topology on \( X \) and \( C \in T_X \) has the property that for \( U \in T \) and \( x \in U \), \( \exists C \in C \) with \( x \in C \subseteq U \Rightarrow C \) is a base for \( Y \).
Proof of Theorem: We show that $B_{x}y$ satisfies the conditions for $C_{x}y$.

In the lemma, say $W$ is open in the topology base $\mathcal{B}$, i.e., for $W$, $x \in W$ with $(x, y) \in U \times V \subseteq W$.

By definition of the product topology base $\mathcal{B}$, for any $y$ with $x \in B \subseteq \mathcal{B}$, there exists $y \in y$.

But $Y_{x}$ has base $B_{x}$, so for any $x \in B_{x}$, $y \in C \subseteq V$ with $x \in B \subseteq C$. Similarly, for any $y \in C$, there exists $B_{x}$ with $y \in B_{x}$.

So $B_{x}y$ satisfies the lemma.
But $\text{R} \times \text{I} \times \text{s}$ yields $\text{R} \times \text{I}$

Therefore, $\exists (a,b) \in (\mathbb{Q}^0)^3$

base for $\text{R} \times \text{I} \times \text{s}$ on $\mathbb{P}^2$.

$\text{R} \times \text{s}$ has base $\mathbb{Z} \cap (0,1,1)$

$\text{R}$ has base $\mathbb{Z} \cap [0,1,0)$

Examine $\text{R} \times \text{I} \times \text{s}$
\[ x \mapsto x, \quad y \mapsto x \mapsto x \]

\[ \begin{align*}
T_1 : & \text{ } \mathbb{R}^2 \rightarrow \mathbb{R}^2 \\
T_1^{-1} : & \text{ } \mathbb{R}^2 \rightarrow \mathbb{R}^2 \\
T_2 : & \text{ } \mathbb{R}^2 \rightarrow \mathbb{R}^2 \\
T_2^{-1} : & \text{ } \mathbb{R}^2 \rightarrow \mathbb{R}^2
\end{align*} \]

\[ T_1 : (x, y) \mapsto (y, x) \]

\[ T_2 : (x, y) \mapsto y \]

\[ T_1^{-1} : (x, y) \mapsto (x, y) \]

\[ T_2^{-1} : (x, y) \mapsto (x, y) \]

\[ \begin{align*}
T_1^{-1} (A) & = A \times y \\
T_2^{-1} (B) & = B \times y \\
T_2 (B) & = B \times x \\
T_1 (A) & = A \times x
\end{align*} \]
Thus if \( u \in \mathcal{F}_x \Rightarrow \Pi^{-1}_1(u) \) is open in \( \mathcal{G}_{x \times y} \),

\( \Pi^{-1}_1(u) = \mathcal{U} \times y \)

Similarly \( v \in \mathcal{G}_y \Rightarrow \Pi^{-1}_2(v) \) is open \( \mathcal{G}_{x \times y} \).

Theorem: \( \exists \ \Pi^{-1}_1(u) : u \in \mathcal{G}_{x \times y} \)

\( \exists \ \Pi^{-1}_2(v) : v \in \mathcal{G}_{y \times z} \)

is a subbase for \( \mathcal{G}_{x \times y} \).
Idea in proof

\[ \pi_1^{-1}(y) \land \pi_2^{-1}(v) \]

\[ = (u \times y) \land (x \times v) \]

\[ = u \times v \text{ a base element for } x \times y. \]