Applications of the Hasse-Weil Bound to Permutation Polynomials over Finite Fields

Xiang-dong Hou
Department of Mathematics and Statistics
University of South Florida, Tampa, FL 33620
xhou@usf.edu

Abstract — A polynomial $f(X,Y) \in \mathbb{F}_q[X,Y]$ is called *absolutely irreducible* if it is irreducible in $\overline{\mathbb{F}}_q[X,Y]$, where $\overline{\mathbb{F}}_q$ is the algebraic closure of $\mathbb{F}_q$. The Hasse-Weil bound provides an estimation for the number of zeros (in $\mathbb{F}_q \times \mathbb{F}_q$) of an absolutely irreducible polynomial in $\mathbb{F}_q[X,Y]$. The Hasse-Weil bound is a consequence of the Riemann's hypothesis for function fields over finite fields and has many applications in the arithmetic of finite fields. In this talk we explore two recent applications where two conjectures about permutation polynomials over $\mathbb{F}_q$ are settled for large q. We will also discuss the techniques in the proofs of the absolute irreducibility of the polynomials involved.