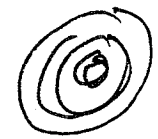
Exam

- In class Review last day of class
  April 24
- No class Mon 22.
  - work some Review prob
    to be ready for Wed.

Stability of nonlinear equilibrium

- NONLIN STAB same as Lin stab except when $\lambda = \pm i\beta$, or $\lambda = 0$.



$$\frac{dx}{dt} = 7x - x^2 - 2xy = x(7 - x - 2y) = f$$ (3)

$$\frac{dy}{dt} = 5y - y^2 - xy = y(5 - y - x) = g$$

(1) Last time, set RHS = 0 get equilibrium $(0,0), (0,5), (7,0)$, and $(3,2)$

(2) $$\nabla \vec{F}(x,y) = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{bmatrix} = \begin{bmatrix} 7-2x-2y & -2x \\ -y & 5-2y-x \end{bmatrix}$$

(3) $(0,0)$: $$\nabla F(0,0) = \begin{bmatrix} 7 & 0 \\ 0 & 5 \end{bmatrix}$$ $\lambda = 5, 7$

Lin: unstab, sour

NONLIN: Same

- $(0,5)$ $\nabla F(0,5) = \begin{bmatrix} 7-10 & 0 \\ -5 & 5-10 \end{bmatrix} = \begin{bmatrix} -3 & 0 \\ -5 & -5 \end{bmatrix}$ (4)

$\lambda = -3, -5$, ~~Saddle~~

Lin: asymp stable, sink

NONLIN: SAME.

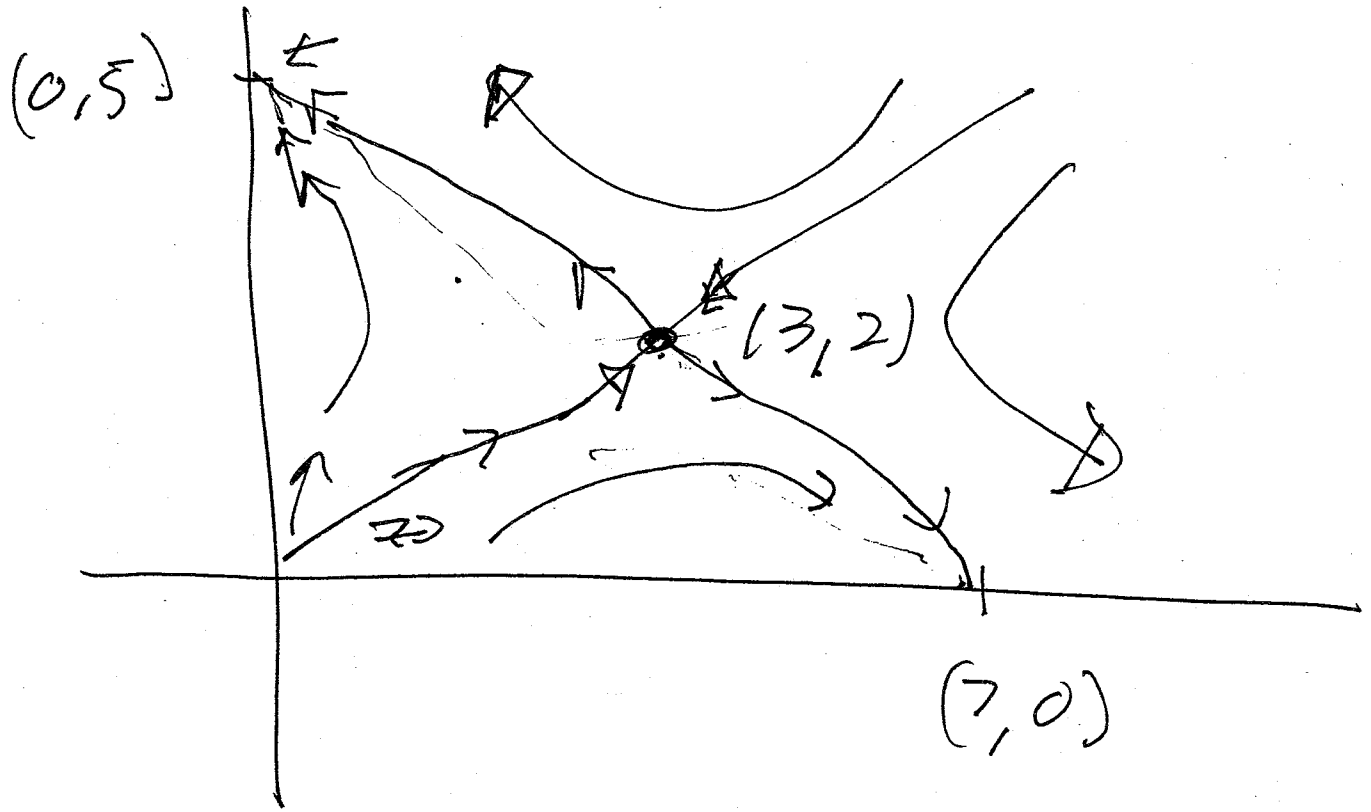
- $(7,0)$, $\nabla F(7,0) = \begin{bmatrix} -7 & -14 \\ 0 & -2 \end{bmatrix}$

$\lambda = ~~-3, 5~~ -7, -2$

Lin: asymp stable, sink

NONLIN: SAME.

- $(3,2)$ – LIN + NONLIN SADDLE UNSTAB – Did last time


(0,5)
(3,2)
(7,0)

Stability of equilibrium of 2nd order DE.

Step 0: Turn into 2-dim system.

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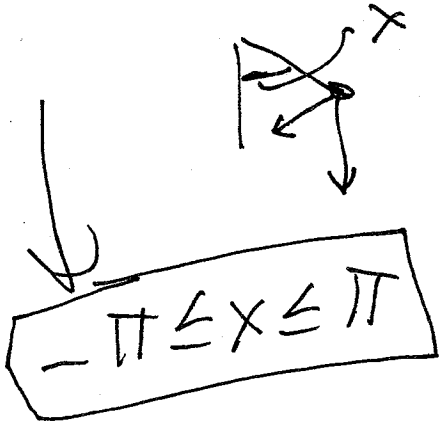
eg 1

$$x'' + 4x' + \sin x = 0$$

$$x'' = -4x' - \sin x$$

$$x' = v$$

$$v' = x'' = -4x' - \sin x = -4v - \sin x$$



$$\frac{dx}{dt} = v = 0$$

$$\frac{dv}{dt} = -4v - \sin x = 0$$

(1) equilibria. $v = 0$

$$-4v - \sin x = 0$$

$v = 0$, $\sin x = 0$ $x = 0, \pm\pi$

3 equilis $(0,0)$ $(\pi,0)$ $(-\pi,0)$.

(2) $\nabla \vec{F}(x,v) = \begin{bmatrix} 0 & 1 \\ -\cos x & -4 \end{bmatrix}$ $T = -4$

$D = \cos x$

$$\lambda = \frac{-4 \pm \sqrt{16 - 4\cos x}}{2}$$

(3) $(0,0)$, $\frac{-4 \pm \sqrt{12}}{2} = \frac{-4 \pm 2\sqrt{3}}{2}$

$= -2 \pm \sqrt{3}$ $(\sqrt{3} < 2.)$

So $-2 + \sqrt{3} < 0$ } Both $\lambda < 0$
$-2 - \sqrt{3} < 0$ }

asymp

Lin: Stable, stable

Nonlin: Same.

$(0, \pm\pi)$

$$\frac{-4 \pm \sqrt{16+4}}{2}$$

$$= \frac{-4 \pm \sqrt{20}}{2} = \frac{-4 \pm 2\sqrt{5}}{2} = -2 \pm \sqrt{5}$$

$\sqrt{5} > 2.$

$-2 - \sqrt{5} < 0$ } one $\lambda > 0$

$-2 + \sqrt{5} > 0$ } one $\lambda < 0$

~~NO HE~~ LIN: SAD., unstable.

NONLIN: SAME

$$x'' + x - x^4 = 0$$

(0) $x' = v$

$v' = -x'' = -x + x^4$

$x' = v$

$v' = -x + x^4$

(1) $v = 0$, $-x + x^4 = 0 \equiv x(-1 + x^3)$ $x = 0$, $x = 1$

$(0,0)$, $(0,1)$.

(2) $\nabla F(x,v) = \begin{bmatrix} 0 & 1 \\ -1 + 4x^3 & 0 \end{bmatrix}$

$T = 0$

$D = 1 - 4x^3$

$$\lambda = \frac{0 \pm \sqrt{0^2 - 4(1-4\times 3)}}{2}.$$

(3) $(0,0)$, $\lambda = \frac{\pm\sqrt{-4}}{2} = \pm i$

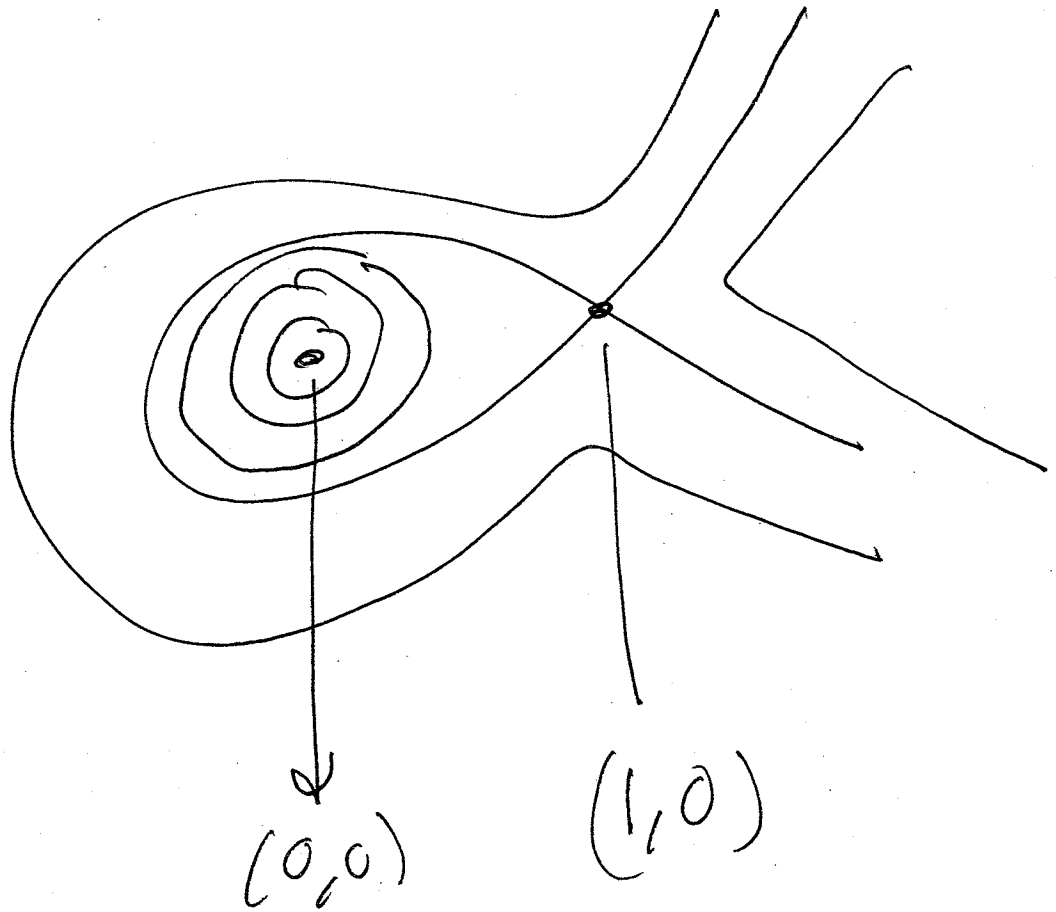
Lin: center, stable

NONLIN: undecided. based on just this info.

$(0,1)$, $\pm\sqrt{12}$ ⑤

Lin: saddle, unstab

NONLIN: SAME.


(0,0)
(1,0)