L1: For each of the following, classify the origin as stable, unstable or asymptotically stable and specify its type as a saddle, sink, source or center. Your answer must be justified by an eigenvalue calculation

(a) $$\frac{d\mathbf{X}}{dt} = \begin{pmatrix} 2 & 13 \\ -1 & -2 \end{pmatrix} \mathbf{X}$$

(b) $$\frac{d\mathbf{X}}{dt} = \begin{pmatrix} 0 & 2 \\ -1 & 3 \end{pmatrix} \mathbf{X}$$

(c) $$\frac{d\mathbf{X}}{dt} = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix} \mathbf{X}$$

(d) $$\frac{d\mathbf{X}}{dt} = \begin{pmatrix} -1 & 1 \\ -1 & -3 \end{pmatrix} \mathbf{X}$$

(e) $$\frac{d\mathbf{X}}{dt} = \begin{pmatrix} -1 & 1 \\ -2 & -4 \end{pmatrix} \mathbf{X}$$

(f) $$\frac{d\mathbf{X}}{dt} = \begin{pmatrix} -1 & -2 \\ 5 & 5 \end{pmatrix} \mathbf{X}$$

L2: Find the equilibrium and classify and specify it as in L1

(a) $$\frac{d\mathbf{X}}{dt} = \begin{pmatrix} -3 & -1 \\ 29 & 1 \end{pmatrix} \mathbf{X} + \begin{pmatrix} -4 \\ 30 \end{pmatrix}$$

(b) $$\frac{d\mathbf{X}}{dt} = \begin{pmatrix} -1 & 1 \\ 0 & +1 \end{pmatrix} \mathbf{X} + \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

L3: Classify the equilibrium at the origin as in L1

(a) $$\frac{dx}{dt} = -2x + 13y + 3xy$$

$$\frac{dy}{dt} = -x + 2y + x^2$$

(b) $$\frac{dx}{dt} = \sin(y - 3x)$$

$$\frac{dy}{dt} = \cos x - e^y$$

L4: Find all equilibrium and classify as in L1

(a) $x'' + x' + \sin x = 0$ $\quad -\pi \le x \le \pi$

(b) $x'' + x - x^3 = 0$

L5: Find all equilibrium and classify as in L1

$$\frac{dx}{dt} = x - 3x^2 - xy$$

$$\frac{dy}{dt} = 2y - y^2 - 4xy$$

PDE 1: For the equation

$$\frac{\partial u}{\partial t} = 3\frac{\partial^2 u}{\partial x^2} \qquad 0 < x < \pi$$

$$\frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0$$

$$u(x,0) = f(x).$$

(a) Find the solution when $f(x) = \sin^2 x$

(b) Find the formal solution when $f(x) = |x|$.

Hint: $|x| \sim \frac{\pi}{2} - \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{\cos((2n-1)x)}{(2n-1)^2}$

PDE 2: For the equation

$$\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2} \qquad -\infty < x < \infty$$

$$u(x,0) = f(x) \qquad -\infty < x < \infty$$

$$\frac{\partial u}{\partial t}(x,0) = g(x) \qquad -\infty < x < \infty$$

(a) Find the soln when $f(x) = \sin x$, $g(x) = \sin(2x)$

(b) Find the soln when $f(x) = e^{-x^2}$, $g(x) = 0$.

PDE 3: For the equation on the disk $0 \le r \le 2$ in polar coordinates

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$$\Delta u = 0 \qquad 0 \le r \le 2, \quad -\pi \le \theta \le \pi$$

$$u(a,\theta) = f(\theta) \qquad -\pi \le \theta \le \pi$$

(a) Find the solution when $f(\theta) = 2\cos\theta - 3\sin 7\theta$

(b) Find the formal soln when $f(\theta) = \theta^2$

Hint: $\theta^2 \sim \dfrac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \dfrac{(-1)^n}{n^2} \cos(n\theta)$