(L1a) $\lambda = \pm 3i$; stable, center

(b) $\lambda = 1, 2$; unstable, source

(c) $\lambda = -1, 4$; unstable, saddle

(d) $\lambda = -2$ double ; [asymp.] stable, sink

(e) $\lambda = -2, -3$; ~~asymp~~ stable, sink ← corrected 4/21

(f) $\lambda = ~~+~~2 \pm i$; unstable, source ~~asymp stable, sink~~

L2(a) $X_0 = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$ $\lambda = -1 \pm 8i$ asymp stable, sink

corrected 4/24 →

(b) $X_0 = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$, $\lambda = \pm 1$ unstable, saddle.

L3(a) $\lambda = \pm 3i$, linear center, nonlinear undecided

(b) $\lambda = -1, -3$, linear sink, asymp stable so also nonlin

Corrected 4/24

L4(a) equation becomes $x' = v$, $v' = -v - \sin x$. Equib at $(0,0)$, $(-\pi, 0)$, $(\pi, 0)$ ~~$(0,-\pi)$, $(0,\pi)$~~ at $(0,0)$ $\lambda = (-1 \pm \sqrt{3}i)/2$

so lin sink, asymp stable, so also nonlin.

at $(\pm\pi, 0)$ ~~$(0, \pm\pi)$~~, $\lambda = (-1 \pm \sqrt{5})/2$ lin. saddle, unstab so also nonlin.

(b) equation becomes $x' = v$, $v' = -x + x^3$. Equib at $(0,0)$ and $(\pm 1, 0)$ ~~$(0, \pm 1)$~~. At $(0,0)$ $\lambda = \pm i$ so lin center, stab but nonlin. undecided. At $(\pm 1, 0)$ ~~$(0, \pm 1)$~~, $\lambda = \pm\sqrt{2}$ so lin saddle, unstab so nonlin also.

L5: Equilib at $(0,0)$, $(0,2)$, $(\frac{1}{3},0)$, $(1,-2)$

- $(0,0)$ — $\lambda = 1, 2$ Lin. source, unstab so nonlin also
- $(0,2)$ — $\lambda = -1, -2$, Lin. sink, asymp stab so nonlin also
- $(1/3, 0)$ — $\lambda = -1, 2/3$, Lin. Saddle, unstab so nonlin also
- $(1,-2)$ — $\lambda = \frac{-1 \pm \sqrt{7}\, i}{2}$, Lin sink, asymp stab so nonlin also

PDE 1

(a) $\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x \quad \beta = 3$

$$u(x,t) = \frac{1}{2} - \frac{1}{2} e^{-12t} \cos 2x$$

(b) $$u(x,t) = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{\infty} e^{-3(2n-1)^2 t} \frac{\cos((2n-1)x)}{(2n-1)^2}$$

PDE 2

(a) $$\frac{1}{2}\left[\sin(x+2t) + \sin(x-2t)\right] - \frac{1}{8}\left[\cos(2x+4t) - \cos(2x-4t)\right]$$

(b) $$\frac{1}{2}\left[e^{-(x+2t)^2} + e^{-(x-2t)^2}\right]$$

PDE 3

(a) $$u(r,\theta) = 2\left(\frac{r}{2}\right)\cos\theta - 3\left(\frac{r}{2}\right)^7 \sin 7\theta$$

(b) $$u(r,\theta) = \frac{\pi^2}{2} + 4\sum_{n=1}^{\infty}\left(\frac{r}{2}\right)^n \frac{(-1)^n}{n^2}\cos(n\theta)$$