MAA4402 • FALL 2017 • SAMPLE EXAM # 1 • PROF. BOYLAND

Justify all answers! Show all work! You will get no credit for just the answer. All problems have the same point value

1. Show that

$$f(x+iy) = -y - e^{-y}\sin(x) + i\left(x + e^{-y}\cos(x)\right)$$

is an entire function.

2. Find all values of $i^{4/5}$. Which is the principle value? Show that one of the values of $i^{4/5}$ is equal to 1.

3. Compute $\log(-1-i)$ and $\mathrm{Log}(-1-i)$.

4. Show using the definitions in terms of exponentials,

$$\cosh(2z) = \cosh^2(z) + \sinh^2(z).$$

5. Compute the principal value of $i^{\,2-i}$ and put the answer in rectangular form.

6. Let C be the unit circle $|z| = 1$ oriented counter-clockwise. Compute

$$\oint_C \overline{z}^{\,2}\, dz.$$

7. Show that for all z,

$$|z+i|^2 - |z-i|^2 = 4\,\mathrm{Im}(z).$$

8. Using the antiderivative,

$$\int_\gamma z\sin(z^2)\, dz,$$

where γ is a path from $(\pi/2 + i\ln(2))$ to $(\pi/2 - i\ln(2))$. You must put the answer in rectangular form.