. $G(x,s) = \begin{cases} -(s^2 - s^{-2})(x^2 - 16x^{-2})/60, & 1 \le s \le x, \\ -(x^2 - x^{-2})(s^2 - 16s^{-2})/60, & x \le s \le 2, \end{cases}$

and

$y = [16x^2 \ln 2 - 16x^{-2} \ln 2 - 15x^2 \ln x]/60$

1. $G(x,s) = \begin{cases} -s(x-1), & 0 \le s \le x, \\ -x(s-1), & x \le s \le 1, \end{cases}$ and

$$y = \alpha + (\beta - \alpha)x - x\int_0^1 sf(s)\,ds + \int_0^x sf(s)\,ds + x\int_x^1 f(s)\,ds$$

23. (a) $K(x,s) = \begin{cases} -(e^{-s} - e^{-2s})(e^{-x} - e^{1-2x})/[(e-1)e^{-3s}], \\ \qquad 0 \le s \le x, \\ -(e^{-x} - e^{-2x})(e^{-s} - e^{1-2s})/[(e-1)e^{-3s}], \\ \qquad x \le s \le 1 \end{cases}$

(b) $y = (1/4)(3 - e^2)e^{-x}/(e-1) + (1/4)e(e-3)e^{-2x}/(e-1) - (1/2)x + (3/4)$

25. (a) $K(x,s) = \begin{cases} -x(2-x)(s^{-3} - s^{-2}), & 1 \le s \le x, \\ -x(1-x)(2s^{-3} - s^{-2}), & x \le s \le 2 \end{cases}$

(b) $y = x^2 \ln 2 - x \ln x - x \ln 2$

27. $H(x,s) = \begin{cases} -[(x^3/2 - \pi x^2)/\pi^2 + x/2]s^2 \\ -[(-x^3/3 + \pi x^2/2)/\pi^3 - 1/6]s^3, \\ \qquad 0 \le s \le x, \\ -[(s^3/2 - \pi s^2)/\pi^2 + s/2]x^2 \\ -[(-s^3/3 + \pi s^2/2)/\pi^3 - 1/6]x^3, \\ \qquad x \le s \le \pi \end{cases}$

29. $H(x,s) = \begin{cases} s^2(s-3x)/6, & 0 \le s \le x, \\ x^2(x-3s)/6, & x \le s \le \pi \end{cases}$

Exercises 11.7, page 691

1. $\sum_{n=1}^{\infty} b_n J_2(\alpha_{2n}x)$, where $\{\alpha_{2n}\}$ is the increasing sequence of real zeros of J_2 and

$$b_n = \frac{\int_0^1 f(x) J_2(\alpha_{2n}x)\,dx}{(\mu - \alpha_{2n}^2)\int_0^1 J_2^2(\alpha_{2n}x)x\,dx}.$$

3. $\sum_{n=1}^{\infty} a_n J_0(\alpha_{1n}x)$, where $\{\alpha_{1n}\}$ is the increasing sequence of real zeros of J_0', which are also the zeros of J_1, and

$$a_n = \frac{\int_0^1 f(x) J_0(\alpha_{1n}x)\,dx}{(\mu - \alpha_{1n}^2)\int_0^1 J_0^2(\alpha_{1n}x)x\,dx}.$$

5. $\sum_{n=0}^{\infty} a_{2n+1}P_{2n+1}(x)$, where

$$a_{2n+1} = \frac{\int_0^1 f(x)P_{2n+1}(x)\,dx}{[\mu - (2n+1)(2n+2)]\int_0^1 P_{2n+1}^2(x)\,dx}.$$

15. (c) $\sum_{n=0}^{\infty} a_n T_n(x)$, where

$$a_n = \frac{\int_{-1}^1 f(x)T_n(x)\,dx}{(\mu - n^2)\int_{-1}^1 T_n^2(x)(1-x^2)^{-1/2}\,dx}.$$

Exercises 11.8, page 702

1. $\phi(x) \equiv 0$ 9. $\pi/2$

11. Between $\pi\sqrt{1/(\lambda+1)}$ and $\pi\sqrt{6/(\lambda+e^{-5})}$

Review Problems, page 705

1. (a) $\lambda_n = 9 + n^2\pi^2$, $n = 1, 2, 3, \ldots$; and $y_n = c_n e^{-3x}\sin(n\pi x)$, $n = 1, 2, 3, \ldots$, where c_n is any real number.

(b) $\lambda_n = \mu_n^2$, $n = 1, 2, 3, \ldots$, where $\tan \mu_n \pi = -2\mu_n$; and $y_n = c_n \sin \mu_n x$, $n = 1, 2, 3, \ldots$, where c_n is any real number.

3. (a) $\lambda_n = (n - 1/2)^2$, $n = 1, 2, 3, \ldots$, and $y_n = \sqrt{2/\pi}\cos[(n - 1/2)x]$, $n = 1, 2, 3, \ldots$

(b) $\frac{2}{\pi}\sum_{n=1}^{\infty}\left[\frac{(-1)^{n-1}(1-\pi)}{(n-1/2)} + \frac{1}{(n-1/2)^2}\right] \times \cos[(n-1/2)x]$

5. (a) $\int_0^{2\pi} h(x)e^{-3x}\sin 2x\,dx = 0$

(b) $\int_1^2 h(x)\,dx = 0$

7. (a) $G(x,s) = \begin{cases} s, & 0 \le s \le x, \\ x, & x \le s \le 1; \end{cases}$

$y = x^3/6 + x^2/2 - 3x/2$

(b) $G(x,s) = \begin{cases} -(e^s + e^{-s})(e^x - e^{2-x})/(2e^2 + 2), \\ \qquad 0 \le s \le x, \\ -(e^x + e^{-x})(e^s - e^{2-s})/(2e^2 + 2), \\ \qquad x \le s \le 1; \end{cases}$

$y = 4(e^{1+x} + e^{1-x} - 1 - e^2)/(e^2 + 1)$

9. π

CHAPTER 12

Exercises 12.2, page 728

1. Unstable saddle point
3. Asymptotically stable spiral point
5. Asymptotically stable improper node
7. $(3, 2)$ is an asymptotically stable improper node.
9. $(-4, -1)$ is a stable center.
11. $(3, -5)$ is an asymptotically stable improper node.

13. Asymptotically stable improper node

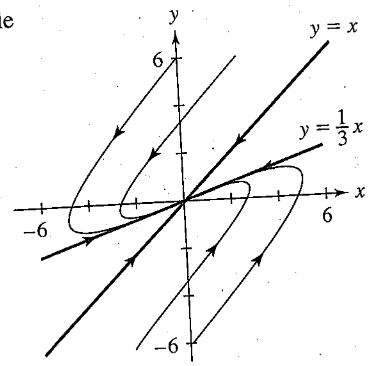

Figure B.46

15. Unstable saddle point

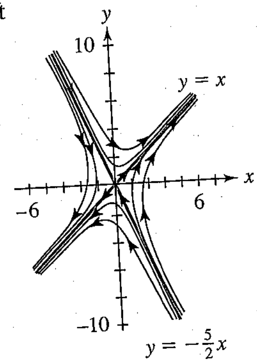

Figure B.47

17. Unstable saddle point

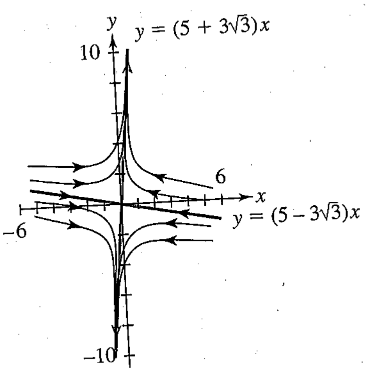

Figure B.48

19. Asymptotically stable improper node

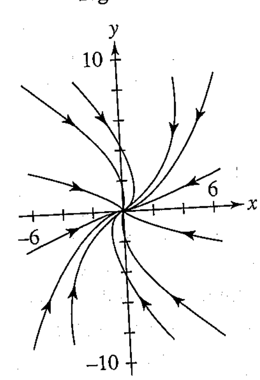

Figure B.49

27. Eigenvalues 3, −1 correspond to eigenvectors [3
and $[1\ 2]^T$ respectively. Solution rays on $y = 2x$
to infinity; solution rays on $y = 2x$ approach the

Exercises 12.3, page 739

1. Unstable saddle point **3.** Unstable improper
5. Center or spiral point of indeterminate stability
7. Unstable saddle point
9. $(8, 2)$ is an asymptotically stable improper node; $(-8, -2)$ is an unstable saddle point.
11. $(2, -2)$ is an unstable improper node; $(3, -3)$ is an unstable saddle point.
13. Critical points $x = (2n + 1)\pi/2$, $x' = 0$ are unstab saddles for all odd n, inconclusive for even n.
15. $x = x' = 0$ is an unstable spiral.
17. $(1, 1)$ is an asymptotically stable improper node, pr node, or spiral point; $(-1, -1)$ is an unstable saddl point. (See Figure B.50.)

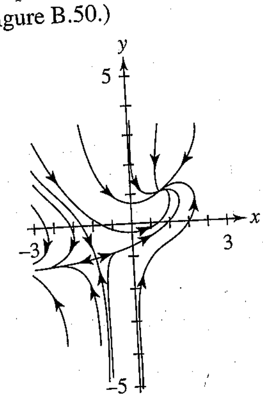

Figure B.50

19. $(0, 0)$ is an unstable improper node, proper node, spiral point; $(0, 3)$ and $(3, 0)$ are unstable saddle and $(2, 2)$ is an asymptotically stable improper n (See Figure B.51.)

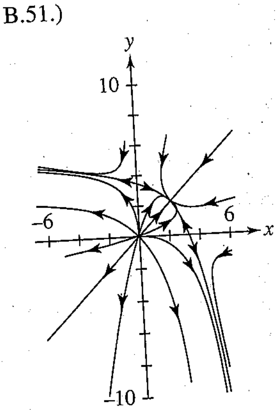

Figure B.51

21. (b) $\varepsilon < -2$, asymptotically stable improper node;
$\varepsilon = -2$, asymptotically stable improper node, proper node, or spiral point;
$-2 < \varepsilon < 0$, asymptotically stable spiral point;
$\varepsilon = 0$, stable center;
$0 < \varepsilon < 2$, unstable spiral point;
$\varepsilon = 2$, unstable improper node, proper node, or spiral point;
$\varepsilon > 2$, unstable improper node

23. (b) a_2/b_2 **(d)** As a_1 is decreased, the population of species x approaches zero (reaches zero when $a_1/c_1 = a_2/b_2$) and the population of species y approaches a_2/b_2.

25. $h = 0$: $(0, 0)$ is an unstable improper node. See Fig. B.52;
$(0, 1/2)$ is an unstable saddle point;
$(1/4, 0)$ is an asymptotically stable improper node;
$(1/3, -1/3)$ is an unstable saddle point, but not of interest.

Species x survives while species y dies off.

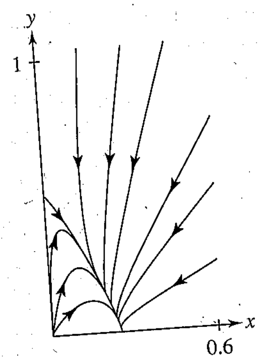

Figure B.52

$h = 1/32$: $((2-\sqrt{2})/16, 0)$ is an unstable improper node;
$((2+\sqrt{2})/16, 0)$ is an asymptotically stable improper node;
$(1/12, 7/24)$ is an unstable saddle point;
$(1/4, -1/8)$ is an unstable saddle point, but not of interest.

Competitive exclusion: One survives, but the other dies off. (See Figure B.53.)

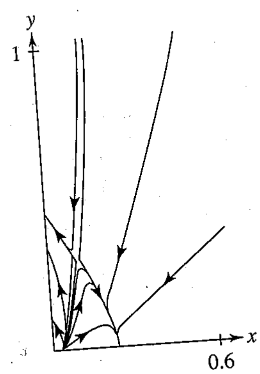

Figure B.53

$h = 5/32$: There are no critical points. Species y survives while species x dies off. (See Figure B.54.)

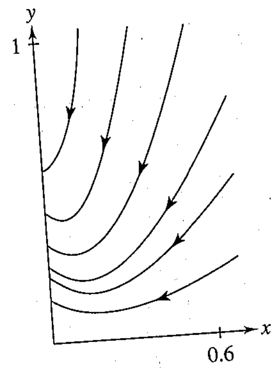

Figure B.54

Exercises 12.4, page 747

1. $G(x) = (1/3)x^3 - (3/2)x^2 + x + C$;
$E(x, v) = (1/2)v^2 + (1/3)x^3 - (3/2)x^2 + x$

3. $G(x) = (1/2)x^2 + x + \ln|x-1| + C$;
$E(x, v) = (1/2)v^2 + (1/2)x^2 + x + \ln|1-x|$

5. $G(x) = (1/4)x^4 + (1/3)x^3 - (1/2)x^2 + C$;
$E(x, v) = (1/2)v^2 + (1/4)x^4 + (1/3)x^3 - (1/2)x^2$

7.

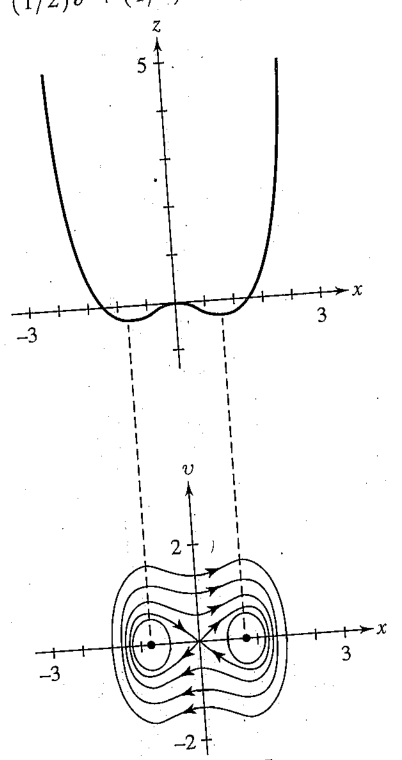

Figure B.55