

$$A = \begin{bmatrix} 2 & 1 & 1 & 0 \\ 4 & 3 & 3 & 1 \\ 8 & 7 & 9 & 5 \\ 6 & 7 & 9 & 8 \end{bmatrix}$$

$$\begin{array}{l} -2R_1 + R_2 \rightarrow R_2 \\ -4R_1 + R_3 \rightarrow R_3 \\ \hline -3R_1 + R_4 \rightarrow R_4 \end{array}$$

$$\begin{bmatrix} 2 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 3 & 5 & 5 \\ 0 & 4 & 6 & 8 \end{bmatrix}$$

OR

$$\begin{bmatrix} 2 & 1 & 1 & 0 \\ 4 & 3 & 3 & 1 \\ 8 & 7 & 9 & 5 \\ 6 & 7 & 9 & 8 \end{bmatrix} =$$

~~$$\begin{bmatrix} 2 & 1 & 1 & 0 \\ 4 & 3 & 3 & 1 \\ 8 & 7 & 9 & 5 \\ 6 & 7 & 9 & 8 \end{bmatrix}$$~~

$$+ \begin{bmatrix} 1 \cdot [2 \ 1 \ 1 \ 0] \\ 2 \cdot [2 \ 1 \ 1 \ 0] \\ 4 \cdot [2 \ 1 \ 1 \ 0] \\ 3 \cdot [2 \ 1 \ 1 \ 0] \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 3 & 5 & 5 \\ 0 & 4 & 6 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 3 & 5 & 5 \\ 0 & 4 & 6 & 8 \end{bmatrix}$$

$$\begin{array}{l} -3R_2 + R_3 \rightarrow R_3 \\ \hline -4R_2 + R_4 \rightarrow R_4 \end{array}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 4 \\ 3 \end{bmatrix} + [2 \ 1 \ 1 \ 0]$$

outer product

$$\begin{aligned}
 &= \begin{bmatrix} 1 \\ 2 \\ 4 \\ 3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 3 \\ 4 \end{bmatrix} + [0111] + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 2 & 4 \end{bmatrix} \xrightarrow{-2R3+R4} \Rightarrow \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 2 & 4 \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 &= \begin{bmatrix} 1 \\ 2 \\ 4 \\ 3 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \\ 3 \\ 4 \end{bmatrix} + [0111] + \begin{bmatrix} 0 \\ 0 \\ -1 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 &= \begin{bmatrix} 2 \\ 2 \\ 4 \\ 3 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \\ 3 \\ 4 \end{bmatrix} + [0111] + \begin{bmatrix} 0 \\ 0 \\ -1 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2 \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 &= \begin{bmatrix} 2 & 2 & 4 & 3 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 1 & 2 \\ 2 & 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = LU
 \end{aligned}$$