Fourier Series

(1) Let $f(t) = \begin{cases} -1 & -\pi \le t \le 0 \\ 1 & 0 < t \le \pi \end{cases}$

(a) sketch the graph of f
(b) compute the orthogonal F. series of f
(c) compute the orthonormal F. series of f
(d) write down the explicit sum formula for the order N least squares truncation error

(2) Let $f(t) = \begin{cases} 0 & -\pi \le t \le 0 \\ t & 0 < t \le \pi \end{cases}$

(a) sketch the graph of f
(b) Find the complex orthonormal F. series of f

(3) Let $f(t) = \begin{cases} 0 & 0 \le t \le \pi/2 \\ 1 & \frac{\pi}{2} < t \le \pi \end{cases}$

(a) sketch the graph of f
(b) compute the sine series of f
(c) compute the cosine series of f.

DFT

(1) If $\omega = e^{2\pi i/N}$ show that for any $0 < q < N$,

$1 + \omega^q + \omega^{2q} + \cdots + \omega^{(N-1)q} = 0$

(2) If $DFT(\vec{x}) = (1, -1, 2, 4)$ and $DFT(\vec{y}) = (3, 2, 1, 7)$. Find

$DFT(\vec{x} * \vec{y})$

(3) If $\vec{x}$ is real and $\overline{X} = DFT(\vec{x})$, show that

$\overline{X}_{N-j} = \overline{X}_j$ for $0 < j < N$

(4) When $N = 4$, write down the matrix F with $DFT(\vec{x}) = \frac{1}{4} F \vec{x}$.

(5) Give a matrix H so that

$\vec{g} * \vec{h} = H\vec{g}$ when $\vec{h} = (-1, 2, 3, -2)$

(6) $\vec{x} = (1, 1, \ldots, 1)$ (all ones). Find

$DFT(\vec{x})$

(7) Show that at the evaluation point $t_n = \frac{n}{N}$

$e^{2\pi i k t} = \overline{e^{2\pi i (N-k) t}}$