Read all these instructions before starting. After you have read them, turn the page and start timing your exam.

1. You have 1.5 hours to do the exam. This does not include the time that it might take you to scan your soln and email them to me.

2. The exam is open book and notes. You may use a calculator or computer, but you must show all steps as if you are doing it by hand so electronic assists may slow you down.

3. Set aside 1.5 hours where you can work undisturbed. During this time you cannot talk to anyone or consult any web resources other than those for this course.

4. The questions sheets are a list of the questions and there is no place to put your answers, so write your answers on separate pieces of paper. You can print the question sheet, or look at it on-screen, but do not include them in your scanned soln.

5. Keep in mind that I have to grade these, so please write neatly, organize your answers, and have your solutions in the same order as the questions.

6. There are 9 (nine) questions.

7. You are bound to all the stated conditions for the exam by the UF Honor code.

8. Your scanned solutions must contain the statement:

   "I have spent at most 1.5 hours working the exam and while I was taking the exam I have consulted no one nor any web resources other than the web pages for this course"

   Then your signature and UF ID number.

Justify all answers! Show all work for partial credit! You will get no credit for just the answer. Different problems have different values. There are 9 (nine) questions

1. *(3 points)* State the Spectral Theorem.

2. *(7 points)* Derive the normal equations for the least squares solution to $A\vec{x} = \vec{b}$ when A is $(m \times n)$, $m \geq n$, and $\text{rank}(A) = n$.

3. *(5 points)* A is $(m \times n)$, $m \geq n$, and $\text{rank}(A) = n$, show using the SVD that $A^T A$ is invertible by giving a formula for its inverse.

4. *(3 points)* If $\text{rank}(A) = r$, how many nonzero singular values does A have?

5. *(3 points)* If the eigenvalues of $A^T A$ are $\lambda_1, \ldots, \lambda_k$, what are the singular values of A?

6. *(7 points)* Let A be a full rank $(m \times n)$-matrix with $m \geq n$. Let the thin QR-decomposition for A be $A = \hat{Q}\hat{R}$. Derive a formula for the pseudoinverse A^+ using $\hat{Q}$ and $\hat{R}$

7. Assume A has the thin SVD:

$$A = \begin{pmatrix} \frac{1}{\sqrt{10}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{2}{\sqrt{10}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{10}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{2}{\sqrt{10}} & 0 & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 10\sqrt{10} & 0 & 0 \\ 0 & 5\sqrt{2} & 0 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} \frac{3}{5} & -\frac{4}{5} & 0 \\ \frac{4}{5} & \frac{3}{5} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

   (a) *(15 points)* Find the best rank one approximation to A in the 2-norm and the best rank two approximation. Your answers must be a fully computed matrices.

   (b) *(5 points)* What are the eigenvalues of the matrix $A^T A$?

8. Let

$$A = \begin{pmatrix} 1 & 1 \\ 2 & -2 \end{pmatrix}$$

   (a) *(15 points)* Find the SVD of A.

   (b) *(5 points)* What is the two-norm condition number of A?

Question 9 is on the next page

9. Let

$$A = \begin{pmatrix} 2 & 2 \\ 2 & 4 \\ 2 & 2 \\ 2 & 4 \end{pmatrix}$$

(a) *(15 points)* Find the thin QR decomposition of A.

(b) *(9 points)* Using part (a) find the least squares solution to

$$A\vec{x} = \begin{pmatrix} 2 \\ 4 \\ 4 \\ -2 \end{pmatrix}$$

(c) *(8 points)* Find the matrix P so that for any vector $\vec{y}$, $P\vec{y}$ is the orthogonal projection onto the column space of A.