Singularities of solutions of the Hamilton-Jacobi equation.
A toy model: distance to a closed subset.

by
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This is a joint work with Piermarco Cannarsa and Wei Cheng.

If A is a closed subset of the Euclidean space $\mathbf{R}^k$, the Euclidean distance function $d_A : \mathbf{R}^k \to [0, +\infty[$ is defined by

$$d_A(x) = \min_{a \in A} \|x - a\|.$$

This function is Lipschitz, therefore differentiable almost everywhere. We will give topological properties of the set $\mathrm{Sing}(F)$ of points in $\mathbf{R}^k \setminus M$ where F is not differentiable. For example it is locally connected. We will also discuss the homotopy type of $\mathrm{Sing}(F)$.

Although, we will concentrate on d_A, we will explain that it is a particular case of a more general result on the singularities of a viscosity solution $U : \mathbf{R}^k \times]0, +\infty[\to \mathbf{R}$ of the evolution Hamilton-Jacobi equation

$$\partial_t U + H(x, \partial_x U) = 0,$$

where $H : \mathbf{R}^k \times \mathbf{R}^k \to \mathbf{R}, (x, p) \mapsto H(x, p)$ is a C^2 Tonelli Hamiltonian, i.e. convex and superlinear in the momentum p.

If time permits we will explain the methods of proof for the case of d_A.