A is an $M \times N$ data array. Recall $\hat{A} = \mathrm{DFT2}(A)$ is the $M \times N$ array given for $m = 0, \ldots, M-1, n = 0, \ldots, N-1$ by

$$\hat{A}_{m,n} = \frac{1}{MN} \sum_{k=0}^{M-1} \sum_{\ell=0}^{N-1} A_{k,l}\, \omega_M^{-km} \omega_N^{-\ell n}$$

where

$$\omega_M = e^{2\pi i/M} \quad \omega_N = e^{2\pi i/N}.$$

1. Assume that the data array A contains just real numbers. Show that for $0 < m < M, 0 < n < N$ that

$$\hat{A}_{M-m,N-n} = \overline{\hat{A}_{M-m,N-n}}$$

.

2. Compute (by hand) $\hat{A}$ for

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}.$$

Note that this is easier using the matrix version of DFT2 we derived in class.

3. Assume the data array A is separable in the sense that there is a M-dimensional vector $\vec{g}$ and an N-dimensional vector $\vec{h}$ so that $A_{m,n} = g_m h_n$ for all m, n. Show that $\hat{A}_{m,n} = \hat{g}_m \hat{h}_n$ for all m, n where $\hat{g} = \mathrm{DFT1}(\vec{g})$ and $\hat{h} = \mathrm{DFT1}(\vec{h})$.

4. For M-dimensional column vector $\vec{g}$ and an N-dimensional column vector $\vec{h}$ their *outer product* is the $M \times N$ matrix A with $A_{m,n} = g_m h_n$, so more succinctly, $A = \vec{g}\,\vec{h}^T$, with the T indicating transpose. Using the result of the previous question, show that if $A = \vec{g}\,\vec{h}^T$ then $\hat{A} = \hat{g}\,\hat{h}^T$