A is an $M \times N$ data array. Recall $\hat{A} = \text{DFT}_2(A)$ is the $M \times N$ array given for $m = 0, \ldots, M-1, n = 0, \ldots, N-1$ by

$$
\hat{A}_{m,n} = \frac{1}{MN} \sum_{k=0}^{M-1} \sum_{\ell=0}^{N-1} A_{k,l} \omega_M^{-km} \omega_N^{-\ell n}
$$

where

$$
\omega_M = e^{2\pi i/M}, \quad \omega_N = e^{2\pi i/N}.
$$

1. Assume that the data array $A$ contains just real numbers. Show that for $0 < m < M, 0 < n < N$ that

$$
\hat{A}_{M-m,N-n} = \overline{\hat{A}_{M-m,N-n}}
$$

2. Compute (by hand) $\hat{A}$ for

$$
A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}.
$$

Note that this is easier using the matrix version of DFT2 we derived in class.

3. Assume the data array $A$ is separable in the sense that there is a $M$-dimensional vector $\vec{g}$ and an $N$-dimensional vector $\vec{h}$ so that $A_{m,n} = g_m h_n$ for all $m, n$. Show that $\hat{A}_{m,n} = \hat{g}_m \hat{h}_n$ for all $m, n$ where $\hat{g} = \text{DFT}_1(\vec{g})$ and $\hat{h} = \text{DFT}_1(\vec{h})$.

4. For $M$-dimensional column vector $\vec{g}$ and an $N$-dimensional column vector $\vec{h}$ their outer product is the $M \times N$ matrix $A$ with $A_{m,n} = g_m h_n$, so more succinctly, $A = \vec{g} \vec{h}^T$, with the $T$ indicating transpose. Using the result of the previous question, show that if $A = \vec{g} \vec{h}^T$ then $\hat{A} = \hat{g} \hat{h}^T$. 