HW 12 • SPRING 2020 • PROF. BOYLAND

1. Compute the Fourier matrix F_N for $N = 6$. Your entries should be exact computed numbers and not decimals, for example, $-1/2 + i\sqrt{3}/2$.

2. Given $u = [2, 5, 1]$ and $v = [-1, 4, 3]$, by hand compute the cyclic convolution $u * v$. Show all work.

3. Let $N = 5$.

   (a) Find the impulse response $\vec{g}$ so that for any vector $\vec{f}$,

   $$(\vec{f} * \vec{g})_n = \frac{1}{4} f_{n-1} + \frac{1}{2} f_n + \frac{1}{4} f_{n+1}$$

   so for each n the output is a weighted average of f_n and its nearest neighbors.

   (b) Give the 5×5 matrix M so that for any vector $\vec{f}$ we have $M\vec{f} = \vec{f} * \vec{g}$.

4. For this question you will need access to a sound file in your system. I give an example from matlab, but you can use any sound file accessible to your system. As usual, your answer must include all code and plots. All figures must have labels.

   Given a real data set $x_1, \ldots, x_N$ (using matlab indexing and notation) with N even, let $\hat{x} = \text{fft}(x)$, the DFT. The periodogram is the plot of the frequency versus P where

   $$\begin{aligned} P(1) &= \frac{1}{N^2} |\hat{x}_1|^2 \\ P(j) &= \frac{2}{N^2} |\hat{x}_j|^2 \text{ for } j = 2, \ldots, N/2 \\ P(\frac{N}{2} + 1) &= \frac{1}{N^2} |\hat{x}_{\frac{N}{2}+1}|^2 \end{aligned}$$

   The frequency is the list $[0 : N/2]/TT$ where TT is the total time, namely, $TT = N/Fs$ where Fs is the number of samples per unit time.

   (a) Write a program that given a real data set and sample rate produces the periodogram plot.

   (b) Let

   $$g(t) = 0.7 \sin(2\pi 50t) + 2 \sin(2\pi 20t).$$

   Create a data set by sampling g at 600 points at the rate of 200 per unit time and plot the periodogram and make sure you get the spikes at the correct frequencies.

   (c) Load the gong sample file in matlab with the command `load gong.mat`. This puts two variable in your workspace, namely, `y` containing the data and `Fs` the sample rate. Produce the periodogram plot for the data. What are approximately the two most dominant frequencies?

   (d) If you want to, you can listen to a sound using `sound(y, Fs)`; this works on most but not all matlab installations.