(1a) Let $S$ be all $(x,y)$ with both $x$ and $y$ integers $\equiv (x,y) : x,y \in \mathbb{Z}$

(1b) Let $S$ be the union of the closed first and third quadrants $\equiv (x,y) : x,y \geq 0$

(2a) The short answer is that the set of solutions is exactly $\left(\begin{array}{c} 2 \\ 3 \\ -5 \end{array}\right)$ and in class we showed that for any $v$, $v^\perp$ is a subspace.

(2b) The simplest answer is that $\vec{0} = (0,0,0)$ doesn't solve the equation. Since $\vec{0}$ is in every subspace, this isn't a subspace.

(3) Pick two vectors $\vec{w}_1, \vec{w}_2 \in V^\perp$ and so $\vec{w}_1 \cdot \vec{v} = 0 = \vec{w}_2 \cdot \vec{v}$ for all $\vec{v} \in V$. Then

$$(x_1 \vec{w}_1 + x_2 \vec{w}_2) \cdot \vec{v} = x_1 \vec{w}_1 \cdot \vec{v} + x_2 \vec{w}_2 \cdot \vec{v}$$

$$= 0 + 0 = 0$$

so $x_1 \vec{w}_1 + x_2 \vec{w}_2 \in V^\perp$. So $V^\perp$ is a subspace.

(4) Many examples, here's one:

$$\begin{pmatrix} 1 & -1 & 1 \\ 2 & 2 & -6 \\ 3 & 1 & -5 \\ 4 & 2 & -8 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = 0$$

or

$$\begin{pmatrix} 1 & -1 & 1 \\ 2 & 2 & -6 \\ 3 & 1 & -5 \\ 4 & 2 & -8 \end{pmatrix} \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix} = 0$$