(1a) Let $S$ be all $(x,y)$ with both $x$ and $y$ integers $= \{(x,y) : x,y \in \mathbb{Z}\} = \mathbb{Z}^2$

(1b) Let $S$ be the union of the closed first and third quadrants $= \{(x,y) : x \cdot y \geq 0\}$

(2). (a) The short answer is that the set of soln is exactly $\begin{pmatrix} 2 \\ 3 \\ -5 \end{pmatrix}^{\perp}$ and in class we showed that for any $\vec{v}$, $\vec{v}^{\perp}$ is a subspace

(b) The simplest answer is that $\vec{0} = (0,0,0)$ doesn't solve the equation. Since $\vec{0}$ is in every subspace, this isn't a subspace.

(3) Pick two vectors $\vec{w}, \vec{w}' \in V^{\perp}$ and so $\vec{w} \cdot \vec{v} = 0 = \vec{w}' \cdot \vec{v}$ for all $\vec{v} \in V$. Then

$(\alpha_1 \vec{w} + \alpha_2 \vec{w}') \cdot \vec{v} = \alpha_1 \vec{w} \cdot \vec{v} + \alpha_2 \vec{w}' \cdot \vec{v}$

$= 0 + 0 = 0$ so $\alpha_1 \vec{w} + \alpha_2 \vec{w}' \in V^{\perp}$ so

$V^{\perp}$ is a subspace.

(4) Many examples, here's on

$$\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} + 2\begin{pmatrix} -1 \\ 2 \\ 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ -6 \\ -5 \\ -8 \end{pmatrix} = \vec{0}$$

or

$$\begin{pmatrix} 1 & -1 & 1 \\ 2 & 2 & -6 \\ 3 & 1 & -5 \\ 4 & 2 & -8 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \vec{0}$$