HW 1 • FALL 2019 • PROF. BOYLAND

1. With respect to the inner product on $[-\pi, \pi]$

$$\langle f, g \rangle = \int_{-\pi}^{\pi} f(t)g(t)\, dt$$

show that

(a) $\langle \sin(nt), \cos(mt) \rangle = 0$,

(b) $\| \sin(nt) \|^2 = \pi$ when $n \neq 0$.

2. Compute the Fourier series of the function $\chi_{\pi/4}$ defined on $[-\pi, \pi]$ by

$$\chi_{\pi/4} = \begin{cases} 1 & \text{if } |t| < \pi/4 \\ 0 & \text{if } |t| \geq \pi/4 \end{cases}$$