HW 1 • SPRING 2020 • PROF. BOYLAND

1. (a) Give a subset of the plane, $\mathbb{R}^2$, that is closed under vector addition and subtraction, but not under scalar multiplication.
   (b) Give a subset of the plane, $\mathbb{R}^2$, that is closed under scalar multiplication but not under vector addition and subtraction.
2. (a) Show that the set of all vectors $\vec{x} = [x_1, x_2, x_3]^T$ such that

$$2x_1 + 3x_2 - 5x_3 = 0$$

   is a subspace of $\mathbb{R}^3$.
   (b) Prove or disprove: the set of all vectors $\vec{x} = [x_1, x_2, x_3]^T$ such that

$$2x_1 + 3x_2 - 5x_3 = 7$$

   is a subspace of $\mathbb{R}^3$.
3. If $V \subset \mathbb{R}^n$ is a subspace, define $V^\perp$ as all the vectors in $\mathbb{R}^n$ orthogonal to every vector in V, so

$$V^\perp = \{\vec{x} \in \mathbb{R}^n : \vec{x} \cdot \vec{v} = 0 \text{ for all } \vec{v} \in V\}.$$

   Show that $V^\perp$ is a subspace.
4. Give an example where a linear combination of three nonzero vectors in $\mathbb{R}^4$ is the zero vector. Then write your example in the form $A\vec{x} = \vec{0}$.