

$$(1) \quad A = \begin{bmatrix} 2 & 1 & 1 \\ 4 & -6 & 0 \\ -2 & 7 & 2 \end{bmatrix} \xrightarrow{\substack{-2R_1 + R_2 \rightarrow R_2 \\ R_1 + R_3 \rightarrow R_3}} \begin{bmatrix} 2 & 1 & 1 \\ 0 & -8 & -2 \\ 0 & 8 & 3 \end{bmatrix} \xrightarrow{R_2 + R_3 \rightarrow R_3}$$

$$\begin{bmatrix} 2 & 1 & 1 \\ 0 & -8 & -2 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{so } L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix} \quad U = \begin{bmatrix} 2 & 1 & 1 \\ 0 & -8 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(c) \quad \det(A) = \det L \det U = 1 \cdot (-16) = -16$$

$$(d) \quad LU\vec{x} = \begin{bmatrix} 3 \\ 16 \\ -10 \end{bmatrix} \quad \text{First solve } L\vec{y} = \begin{bmatrix} 3 \\ 16 \\ -10 \end{bmatrix}$$

$$\text{which is } \begin{cases} x_1 = 3 \\ 2x_1 + x_2 = 16 \\ -x_1 - x_2 + x_3 = -10 \end{cases}$$

$$\Rightarrow \begin{cases} x_1 = 3 \\ x_2 = 16 - 2x_1 = 10 \\ x_3 = -10 + x_1 + x_2 = 3 \end{cases}$$

$$\text{now solve } U\vec{x} = \begin{bmatrix} 3 \\ 10 \\ 3 \end{bmatrix}$$

$$\text{which is } \begin{cases} 2x_1 + x_2 + x_3 = 3 \\ -8x_2 + 2x_3 = 10 \\ x_3 = 3 \end{cases}$$

$$\Rightarrow \begin{cases} x_3 = 3 - 2x_1 - x_2 = 1 \\ x_2 = (10 - 2x_3) / -8 = -1 \\ x_3 = 3 \end{cases}$$

$$\text{so soln is } \vec{x} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$$