

1. Let

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 4 & -6 & 0 \\ -2 & 7 & 2 \end{pmatrix}$$

- (a) Compute (by hand) the LU decomposition of A .
- (b) Check your answer by multiplying LU and confirm you get A .
- (c) Using the LU decomposition, what is $\det(A)$?
- (d) Using the LU decomposition, solve

$$A\vec{x} = \begin{pmatrix} 3 \\ 16 \\ -10 \end{pmatrix}$$

2. Your answer must include your code and the results of running it.

- (a) For L an $(n \times n)$ lower triangular matrix whose diagonal elements are all non-zero, write a program using forward substitution that will solve the equation $L\vec{x} = \vec{b}$.
- (b) Create the (5×5) matrix L with $L_{i,j} = i + j^2$ for $i \geq j$ and $L_{i,j} = 0$ otherwise. Create the vector \vec{b} with $b_i = i^2$ for $i = 1, \dots, 5$. Use your program to numerically solve the equation $L\vec{x} = \vec{b}$.
- (c) Verify your answer is correct by multiplying L by \vec{x} numerically and confirm you get \vec{b} .