HW#2

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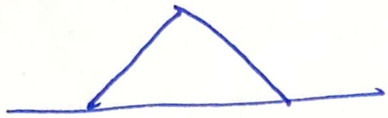
#1



$\boxed{n \neq 0}$

$b_n = 0$

$$a_n = \frac{2}{\pi}\int_0^{\pi} \frac{\pi - t}{2}\cos nt\,dt$$

$$= \frac{1}{\pi}\int_0^{\pi} (\pi - t)\cos nt\,dt$$

$$\boxed{\begin{array}{ll} u = \pi - t & dv = \cos nt\,dt \\ du = -dt & v = \frac{\sin nt}{n} \end{array}} \to = \frac{1}{\pi}\left[(\pi - t)\sin nt\Big|_0^{\pi} + \int_0^{\pi}\frac{\sin nt}{n}\,dt\right]$$

$$= \frac{1}{\pi}\left[-\frac{\cos nt}{n^2}\Big|_0^{\pi}\right] = \frac{1}{\pi}\left[\frac{-\cos n\pi + 1}{n^2}\right]$$

$$= \frac{1}{n^2\pi}\left((-1)^{n+1} + 1\right)$$

$$a_0 = \frac{1}{\pi}\int_0^{\pi}(\pi - t)\,dt = \frac{1}{\pi}\left[\pi t - \frac{t^2}{2}\Big|_0^{\pi}\right]$$

$$= 1/\pi\left[\pi^2 - \frac{\pi^2}{2}\right] = \pi/2$$

$$\boxed{f(t) \sim \frac{\pi}{4} + \sum_{n=1}^{\infty}\frac{1}{n^2\pi}\left((-1)^{n+1} + 1\right)\cos(nt)}$$

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(#2) $T = 2\pi$

even so $b_n = 0$

$n \neq 0 \quad a_n = \frac{1}{2\pi} \int_{-\pi/4}^{\pi/4} \cos\left(\frac{nt}{2}\right) dt$

$= \frac{1}{2\pi} \left. \frac{2\sin\left(\frac{nt}{2}\right)}{n} \right|_{-\pi/4}^{\pi/4}$

$= \frac{1}{2\pi} \frac{4}{n} \sin\left(\frac{n\pi}{8}\right)$

$a_0 = \frac{1}{2\pi} \int_{-\pi/4}^{\pi/4} dt = \frac{1}{2\pi} \frac{\pi}{2} = \frac{1}{4}$

$$\boxed{f(t) \sim \frac{1}{8} + \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin\left(\frac{n\pi}{8}\right) \cos\left(\frac{nt}{2}\right)}$$

#3a odd so $a_n = 0$

$b_n = \frac{2}{\pi} \int_{0}^{\pi} t \sin nt \, dt = \frac{2}{\pi} \left[\left. t\left(-\frac{\cos nt}{n}\right) \right|_0^{\pi} + \int_0^{\pi} \frac{\cos nt}{n} dt \right]$

$u = t \quad dv = \sin nt \, dt$
$du = dt \quad v = -\frac{\cos nt}{n}$

$= \frac{2}{\pi} \left[-\pi \frac{\cos n\pi}{n} + \left. \frac{\sin nt}{n^2} \right|_0^{\pi} \right] = -\frac{2}{n} \cos n\pi = \frac{2}{n}(-1)^{n+1}$

$$\boxed{f(t) \sim \sum_{n=1}^{\infty} \frac{2}{n}(-1)^{n+1} \sin(nt)}$$

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(3b) Just computed $\sum_{n=1}^{\infty} \frac{2}{n}(-1)^{n+1} \sin(nt)$

$= \sum_{n=1}^{\infty} B_n \frac{\sin(nt)}{\sqrt{\pi}}$ orthonormal form.

So $\frac{B_n}{\sqrt{\pi}} = \frac{2}{n}(-1)^{n+1}$ or $B_n = \sqrt{\pi} \cdot \frac{2}{n}(-1)^{n+1}$

So in orthonormal form

$$\boxed{f(t) \sim \sum_{n=1}^{\infty} \left(\sqrt{\pi}\, \frac{2}{n}(-1)^{n+1}\right) \cdot \frac{\sin(nt)}{\sqrt{\pi}}}$$

(3c) $\|E_N\|^2 = \|f\|^2 - \sum_{n=1}^{N} (B_n)^2$

$= \frac{2\pi^3}{3} - \sum_{n=1}^{N} \frac{4\pi}{n^2}$

$$\boxed{\|f\|^2 = \int_{-\pi}^{\pi} t^2\, dt = \frac{t^3}{3}\Big|_{-\pi}^{\pi} = \frac{2\pi^3}{3}}$$