1(a) $\begin{vmatrix} -5-\lambda & -3 \\ 6 & 4-\lambda \end{vmatrix} = (-5-\lambda)(4-\lambda)+18 = \lambda^2+\lambda-2$

$= (\lambda+2)(\lambda-1)$ so $\lambda = 1, -2$

(1b) $\lambda=1$ $\begin{bmatrix} -6 & -3 \\ 6 & 3 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\vec{v} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$\lambda=-2$ $\begin{bmatrix} -3 & -3 \\ 6 & 6 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\vec{v} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

so $X = \begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix}$ yields $X^{-1}AX = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$

(2) $\begin{vmatrix} -2-\lambda & -2 \\ 5 & 4-\lambda \end{vmatrix} = (-2-\lambda)(4-\lambda)+10$
$= \lambda^2-2\lambda+2$

$\lambda = \frac{2 \pm \sqrt{4-8}}{2} = \frac{2\pm 2i}{2} = 1 \pm i$

$\boxed{\lambda = 1+i}$ (a) $-2-(1+i)v_1 + -2v_2 = 0$ or $(3-i)v_1 - 2v_2 = 0$

$\vec{v} = \begin{bmatrix} 2 \\ -3-i \end{bmatrix}$ as one choice.

$\lambda = 1-i$ $-2-(1-i)v_1 - 2v_2 = 0$ or $(-3+i)v_1 - 2v_2 = 0$

$\vec{v} = \begin{bmatrix} 2 \\ -3+i \end{bmatrix}$ is one choice

(3) $\vec{w} \cdot \vec{w} = \left(\sum_{i=1}^{n} \alpha_i \vec{u}_i \right) \cdot \left(\sum_{i=1}^{n} \alpha_i \cdot \vec{u}_i \right)$

$$= \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_i \alpha_j \; \vec{u}_i \cdot \vec{u}_j$$

$$= \sum_{i=1}^{n} \alpha_i \alpha_i \cdot \vec{u}_i \cdot \vec{u}_i$$

$\vec{u}_i \cdot \vec{u}_j = 0$
when $i \neq j$

$$= \sum_{i=1}^{n} \alpha_i^2$$

$\vec{u}_i \cdot \vec{u}_i = \|\vec{u}_i\|^2 = 1.$