1. Let
$$A = \begin{pmatrix} -5 & -3 \\ 6 & 4 \end{pmatrix}$$
   (a) Compute (by hand) the eigenvalues and eigenvectors of A.
   (b) Give a matrix X so that
   $$X^{-1}AX = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$$
   where λ_1 and λ_2 are the eigenvalues you computed in part (a).

2. Let
$$A = \begin{pmatrix} -2 & -2 \\ 5 & 4 \end{pmatrix}$$
   Compute (by hand) the eigenvalues and eigenvectors of A.

3. Assume $\{\vec{u}_1, \dots, \vec{u}_n\}$ is an orthonormal set. If
$$\vec{w} = \sum_{i=1}^{n} \alpha_i \vec{u}_i$$
   show that
$$\|\vec{w}\|^2 = \sum_{i=1}^{n} \alpha_i^2$$

4. Your answer must include your code and the results of running it. You should use built-in or library functions of your system. In Matlab consult the documentation for `rand` and `eig`.
   (a) Create a (4×4)-matrix A whose entries are random numbers between 0 and 1.
   (b) Compute the matrix X and the matrix $D = diag(\lambda_1, \dots, \lambda_4)$ so that
   $$X^{-1}AX = D$$
   and verify this by computing $X^{-1}AX - D$ and seeing that it is (very close to) the zero matrix.