HW3
pg 1

(1) (a) $c_0 = \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} 1\,dt = \frac{1}{2\pi}\cdot\frac{\pi}{2} = \frac{1}{4}$

$n \neq 0 \quad c_n = \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} e^{-int}\,dt = \frac{1}{2\pi}\left.\frac{e^{-int}}{-in}\right|_{-\pi/4}^{\pi/4}$

$= \frac{-1}{2\pi i n}\left[e^{-in\pi/4} - e^{in\pi/4}\right]$

$= -\frac{1}{2\pi i n}\left[\cos n\pi/4 - i\sin n\pi/4 - [\cos n\pi/4 + i\sin n\pi/4]\right]$

$= +\frac{1}{2\pi i n}\cdot 2i\sin n\pi/4 = \frac{1}{\pi n}\sin\frac{n\pi}{4}$

$\chi_{\pi/4}(t) \sim \frac{1}{4} + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{1}{\pi n}\sin\frac{n\pi}{4}\, e^{int}$

(b) $\chi_{\pi/4}(t) = \frac{\sqrt{2\pi}}{4}\cdot\left(\frac{1}{\sqrt{2\pi}}\right) + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{\sqrt{2\pi}}{\pi n}\sin\frac{n\pi}{4}\left(\frac{e^{int}}{\sqrt{2\pi}}\right)$

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#w3
pg2

(2 a) $C_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} t\,dt = \frac{1}{2\pi}\frac{t^2}{2}\Big|_{-\pi}^{\pi} = 0$

$n \neq 0$ $\quad C_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} t\,e^{-int}\,dt = \left[\frac{t\,e^{-int}}{-in}\Big|_{-\pi}^{\pi} + \int_{-\pi}^{\pi}\frac{e^{-int}}{in}\,dt\right]\frac{1}{2\pi}$

$$\boxed{\begin{array}{ll} u = t & dv = e^{-int}\,dt \\ du = dt & v = \frac{e^{-int}}{-in} \end{array}}$$

$$= \left[\frac{\pi e^{-in\pi}}{-in} + \frac{-\pi e^{in\pi}}{in} + \frac{e^{-int}}{n^2}\Big|_{-\pi}^{\pi}\right]\frac{1}{2\pi}$$

$$= \left\{-\frac{\pi}{in}\left[e^{-in\pi} + e^{in\pi}\right] + \frac{1}{n^2}\left[e^{-in\pi} - e^{in\pi}\right]\right]\frac{1}{2\pi}$$

$$= \left[-\frac{\pi}{in}\left[2\cos n\pi\right] - \frac{i}{n^2}\left(-\sin n\pi\right)\right]1/2\pi$$

$$= \pi\frac{i}{n}2(-1)^n \cdot \frac{1}{2\pi} = \frac{i}{n}(-1)^n$$

$$f(t) \sim \sum_{n=1}^{\infty}\frac{i}{n}(-1)^n e^{int}$$

(b) $f(t) \sim \sum_{n=1}^{\infty}\left(\frac{i}{n}(-1)^n\right)\sqrt{2\pi}\,\frac{e^{int}}{\sqrt{2\pi}}$