HW3 – Due Friday, October 28, start of class

No electronic submissions, only hard copy

1. Let $g(z) = 5(1/2)^z + 2(1/3)^z$ for $z \geq 0$.
   (a) Show that $g > 0$, g is decreasing, and $\lim_{z\to\infty} g(z) = 0$.
   (b) Show that there is a unique solution to $g(z) = 1$.
2. Let $h(x) = x^2$ defined for $x \in [0, 1]$.
   (a) Show that h is Lipschitz.
   (b) Show that h is invertible but it is not bi-Lipschitz.
3. Let K_1 be the fractal with $K_1 = f_1(K_1) \cup f_2(K_1)$ for $f_1(x) = (1/3)x$ and $f_2(x) = (1/3)x + 2/3$, and let K_2 be the fractal with $K_2 = g_1(K_2) \cup g_2(K_2)$ for $g_1(x) = (1/5)x$ and $g_2(x) = (1/5)x + 4/5$.
   (a) Is there a bi-Lipschitz map $h : [0, 1] \to [0, 1]$ with $h(K_1) = K_2$? Be sure to justify your answer completely.
   (b) Find *three* distinct similarities α_1, α_2 and α_3 so that $K_1 = \alpha_1(K_1) \cup \alpha_2(K_1) \cup \alpha_3(K_1)$
4. For the iterated function system shown below which defines the fractal K:
   (a) Find similarities f_i so that
   $$K = \bigcup_{i=1}^{5} f_i(K)$$
   (b) Give the equation that the fractal dimension D satisfies and compute D (you can use any software or webpages or calculator for this).