‘ HW 4 • SPRING 2020 • PROF. BOYLAND

1. Let
$$A = \begin{pmatrix} -5 & -3 \\ 6 & 4 \end{pmatrix}$$
Using your work from HW3, solve the differential equation
$$\frac{d\vec{x}}{dt} = A\vec{x} \text{ with } \vec{x}(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$
Give your answer in both the matrix and the vector form.

2. With
$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$$
the trace is $T = A_{11} + A_{22}$ and the determinant is $D = A_{11}A_{22} - A_{12}A_{21}$. Show using the quadratic formula that the eigenvalues of A are
$$\frac{T \pm \sqrt{T^2 - 4D}}{2}$$

3. Let
$$\Phi(\vec{x}) = \vec{a} + \vec{b}^T\vec{x} + \frac{1}{2}\vec{x}^T S\vec{x}$$
where $\vec{a}$ and $\vec{b}$ are given $(n \times 1)$ vectors and S is a symmetric $(n \times n)$ matrix. Give simple formulas for $\nabla\Phi(\vec{x})$ and $H\Phi(\vec{x})$. Be sure to justify your answer.

4. You can use your favorite computational platform for this problem along with any built-in or library functions.
$$\Phi(x_1, x_2) = 10x_1^2x_2 - 5x_1^2 - 4x_2^2 - x_1^4 - 2x_2^4.$$
You are given that $(0,0)$, $(2.6442, 1.8984)$ and $(.8567, .6468)$ are numerical approximations of critical points of Φ.

   (a) Compute $\nabla\Phi$ by hand and then verify numerically that the given points are near critical.

   (b) Compute $H\Phi$ by hand and then using your computational platform classify each point as local min, max, or saddle.