#W4.1

1(a) $a_n = \frac{2}{\pi}\int_0^{\pi} t \sin(nt)\,dt \qquad n \neq 0$

$$\boxed{\begin{matrix} u = t & dv = \sin nt\,dt \\ du = dt & v = -\frac{\cos nt}{n} \end{matrix}}$$

$$= \frac{2}{\pi}\left[t\frac{(-\cos nt)}{n}\Big|_0^{\pi} + \int_0^{\pi} \frac{\cos nt}{n} \right]$$

$$= \frac{2}{\pi}\left[-\frac{\pi\cos n\pi}{n} + \frac{\sin n\pi}{n^2} - \frac{\sin 0}{n^2} \right]$$

$$= \frac{2}{n}(-1)^{n+1}$$

$$\boxed{f(t) \sim \sum_{n=1}^{\infty} \frac{2}{n}(-1)^{n+1} \sin(nt).}$$

(b) $a_0 = \frac{2}{\pi}\int_0^{\pi} t\,dt = \frac{2}{\pi}\frac{\pi^2}{2} = \pi$

$n \neq 0 \quad a_n = \frac{2}{\pi}\int_0^{\pi} t\cos nt\,dt$

$$\boxed{\begin{matrix} u = t & dv = \cos nt\,dt \\ du = dt & v = \frac{\sin nt}{n} \end{matrix}}$$

$$= \frac{2}{\pi}\left[t\frac{\sin nt}{n}\Big|_0^{\pi} - \int_0^{\pi} \frac{\sin nt}{n}\,dt \right]$$

$$= \frac{2}{\pi}\left(\frac{\cos nt}{n^2}\right)\Big|_0^{\pi} = \frac{2}{\pi n^2}\left((-1)^n - 1\right)$$

$$\boxed{f(t) \sim \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{2}{\pi n^2}\left((-1)^n - 1\right)}$$

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(2) (a) $f(t) = \sin 3t$ (already in sine series form)

(b)

$$a_0 = \frac{2}{\pi}\int_0^{\pi} \sin(3t)\,dt$$

$$= \frac{2}{\pi}\, \frac{-\cos(3t)}{3}\Big|_0^{\pi} = \frac{2}{\pi \cdot 3}\left[-\cos(3\pi) + 1\right]$$

$$= \frac{4}{3\pi}$$

$n \neq 0$

$$a_n = \frac{2}{\pi}\int_0^{\pi} \sin(3t)\,\cancel{\sin}\overset{\cos}{}(nt)\,dt$$

$$= \frac{2}{\pi}\frac{1}{2}\int_0^{\pi} \sin((3+n)t) + \sin((3-n)t)\,dt$$

$\boxed{n \neq 3}$

$$= \frac{1}{\pi}\; \frac{-\cos((3+n)t)}{3+n} - \frac{\cos((3-n)t)}{3-n}\Big|_0^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{-(-1)^{3+n}+1}{3+n} + \frac{-(-1)^{3-n}+1}{3-n}\right]$$

$$= \frac{1}{\pi}\left[\frac{(-1)^{n}+1}{3+n} + \frac{(-1)^{n}+1}{3-n}\right]$$

$\underline{\underline{n=3}}$ $a_3 = \frac{1}{\pi}\int_0^{\pi} \sin(6t)\,dt$

$= \frac{1}{\pi} \left. \frac{-\cos(6t)}{6} \right|_0^{\pi}$

$= \frac{1}{\pi} \frac{-\cos(6\pi)+1}{6} = 0$

$f(t) = \frac{2}{3\pi} + \sum_{\substack{n=1 \\ n\neq 3}}^{\infty} \frac{1}{\pi}\left[\frac{(-1)^n+1}{3+n} + \frac{(-1)^n+1}{3-n}\right]\cos(nt)$

$= \frac{2}{3\pi} + \sum_{\substack{n=1 \\ n\neq 3}}^{\infty} \frac{6}{\pi}\frac{((-1)^n+1)}{9-n^2} \cos nt$

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