HW 5 • FALL 2019 • PROF. BOYLAND

For each interger $N > 0$ define an $N \times N$ matrix G_N by its components $(G_N)_{i,j} = \omega^{(i-1)(j-1)}$ for $i = 1, \ldots, N$ and $j = 1, \ldots, N$ where

$$\omega = e^{2\pi i/N}.$$

1. Compute G_N for $N = 6$. Your entries should be exact numbers not decimals. For example, $(G_6)_{6,5} = \omega^{20} = \omega^2 = e^{4\pi i/6} = \cos(2\pi/3) + i\sin(2\pi i/3) = -1/2 + i\sqrt{3}/2$.

2. Show that the last row of G_N for any N can be reduced to

$$1, \omega^{-1}, \omega^{-2}, \ldots, \omega^{-(N-2)}, \omega^{-(N-1)}$$

   and then show this is equal to

$$1, \omega^{N-1}, \omega^{N-2}, \ldots, \omega^2, \omega$$

   and also to

$$1, \overline{\omega}, \overline{\omega}^2, \ldots, \overline{\omega}^{N-2}, \overline{\omega}^{N-1}$$

3. Given $u = [2, 5, 1]$ and $v = [-1, 4, 3]$, by hand compute the cyclic convolution $u * v$. Show all work.