HW5 – Due Monday, November 14, start of class
No electronic submissions, only hard copy

1. Let $X = \{1, 2, 3\} \subset \mathbb{R}$, so X is a three point set inside the real line. Show that its 0-dimensional Hausdorff measure is $\mathcal{H}(X, 0) = 3$ and for any $p > 0$, its p-dimensional Hausdorff measure is $\mathcal{H}(X, p) = 0$. Thus the Hausdorff dimension of X is zero.
2. A function $f : (X, d) \to (Y, \rho)$ is called Hölder if there are constants $C, \nu > 0$ so that
$$\rho(f(x), f(x')) < C\ d(x, x')^\nu$$
for all $x, x' \in X$.
   (a) Show that any Hölder function is continuous.
   (b) Show that if $f, g : (X, d) \to (X, d)$ are a pair of Hölder functions, then their composition $f \circ g$ is also Hölder.
3. Let C be the standard middle third Cantor set and $f : [0, 1] \to [1, 4]$ is defined by $f(x) = (x + 1)^2$. Let
$$C' = f(C) = \{f(x) \colon x \in C\}.$$
Compute the Hausdorff dimension of C' and be sure to justify your answer thoroughly. Hint: remember the theorem on bi-Lipschitz functions.
4. Let $K \subset [0, 1]$ be defined by the two similarities $f_1(x) = (1/4)x$ and $f_2(x) = (1/4)x + 3/4$ with $K = f_1(K) \cup f_2(K)$. Define $\beta : \Sigma_2^+ \to K$ by
$$\beta(\underline{s}) = \bigcap_{n=0}^{\infty} f_{s_0} f_{s_1} \dots f_{s_n}(I).$$
Show that β is a Hölder function.