‘ HW 5 • SPRING 2020 • PROF. BOYLAND

1. Let A have SVD $A = U\Sigma V^T$. What is the SVD of A^T, the SVD of A^TA, and the SVD of A^{-1} where in the last case we assume A is square with all positive singular values?

2. Let
$$A = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{pmatrix}$$
Compute (by hand) the nonzero singular values of A (just these not the whole decomposition).

3. Let
$$A = \begin{pmatrix} 1 & 0 \\ 0 & 2 \\ 1 & 0 \\ 0 & 2 \end{pmatrix}$$
Compute (by hand) the thin or reduced SVD of A. So find matrices with $A = U\Sigma V^T$ with U a 4×2 matrix with orthonormal columns, Σ a 2×2 diagonal matrix and V a 2×2 orthogonal matrix.

4. Your answer must include your code and the results of running it. You should use built-in or library functions of your system.
   (a) For $i = 1, \ldots, 100$ create a (10×10)-matrix A_i whose entries are random numbers between 0 and 1 and let $C_i = \sigma_1/\sigma_{10}$, the ratio of the largest and smallest singular values of A_i.
   (b) Plot i vs C_i.
   (c) For the data set $\{C_1, \ldots, C_{100}\}$ compute the mean and the standard deviation
   (d) The number C_i is the *condition number* of A_i and is a measure of how badly behaved the matrix is under standard numerical methods. We will study it in detail later.