

Fix an even integer N . Recall $\vec{X} = \text{DFT}(\vec{x})$ is defined for $n = 0, \dots, N-1$ by

$$X_n = \frac{1}{N} \sum_{k=0}^{N-1} x_k \omega^{-kn}$$

where

$$\omega = e^{2\pi i/N}.$$

1. Assume that the data vector \vec{x} contains just real numbers.
 - (a) Show that for $0 < m < N$ that $X_{N-m} = \overline{X_m}$.
 - (b) Show that X_0 and $X_{N/2}$ are both real numbers. Give explicit formulas for them that don't use e or trig functions.
2. For some q assume that the data vector is $\vec{x} = (0, 0, \dots, 1, \dots, 0)$ with the 1 in the q^{th} slot. Compute (by hand) $\text{DFT}(\vec{x})$.
3. Now for some q assume that the data vector is $\vec{x} = (1, \omega^q, \omega^{2q}, \dots, \omega^{(N-1)q})$.
 - (a) Use Euler's formula to write \vec{x} in terms of sines and cosines.
 - (b) Compute (by hand) $\text{DFT}(\vec{x})$.